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2023 年 8 月 22 日　Email: shutingnpu@yahoo.co.jp

Vibration Analysis of a Pair of Spur Gears Using a Power-Absorbing Gear Test Equipment

By Prof. Shuting Li

1． Abstract

To reduce gear vibration and noise, it is necessary to be able to accurately analyze gear vibration theoretically. However, it is well known that gear vibration analysis is not straightforward, as gear vibration is influenced by numerous factors: including gear machining accuracy, assembly accuracy, tooth surface finishing, and tooth surface friction. as well as the number of meshing teeth and the meshing position. I am one of many researchers who have conducted research on gear vibration analysis. Since around 1987, I have been exploring analytical methods for gear vibration through my own research and have also been involved in the development of specialized software; however, even after more than 20 years, my research had not progressed significantly. In recent years, there have been major advances in gear vibration analysis, so I would like to briefly introduce the results of that analysis here.

2． Power-Absorbing Gear Test Machine

To verify the validity of gear vibration analysis results, it is necessary to measure gear vibration in practice; for this purpose, the power-absorbing gear test machine shown in Figure 1 can be used. However, when using this test machine to measure the vibration of a pair of gears, care must be taken during measurement because, if the reduction ratio of the pair is low, it is not possible to apply a large torque to the output shaft side of the gears. Figures 1 and 2 show the power-absorbing gear test machine designed and fabricated by the author.

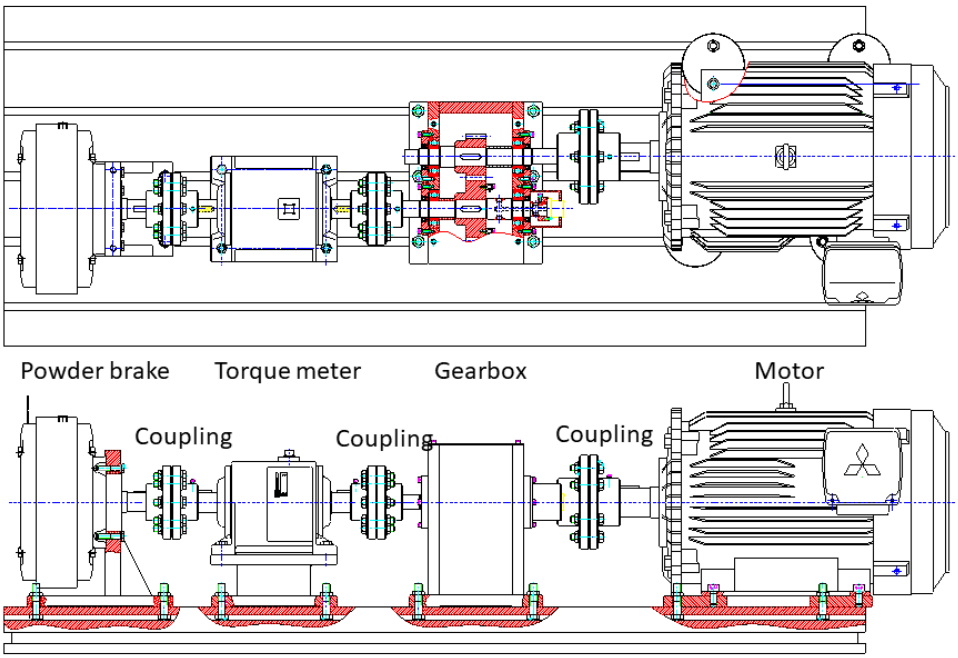

Figure 1: Design drawing of a power-absorbing gear tester

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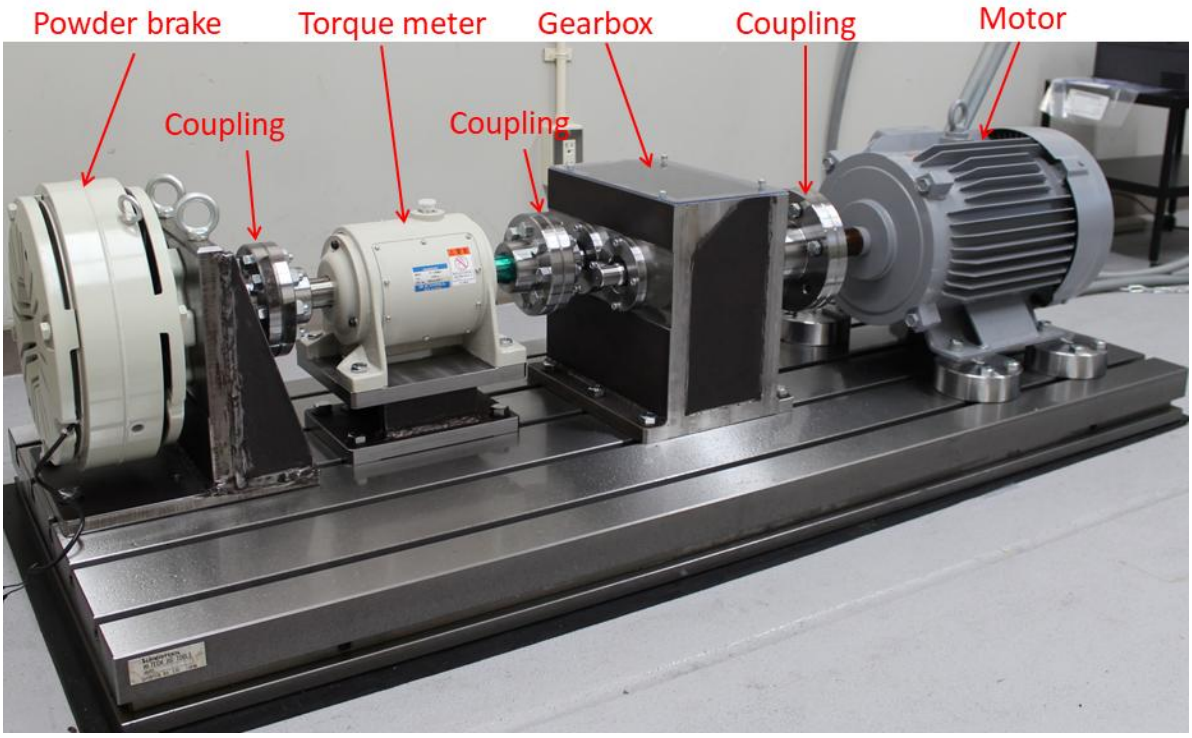

Figure 2: Prototype power-absorbing gear tester

3． Vibration Analysis Modeling

To analyze the torsional vibration in the rotational direction of a pair of spur gears, a vibration analysis model was developed for the power-absorbing gear test machine shown in Figures 1 and 2. First, Figure 3 shows the proposed 7-degree-of-freedom (7-DOF) translational vibration analysis model. This model is a method for studying the torsional vibration of the gears by converting the rotational torsional vibration of the device into translational vibration along the line of action of the gear pair and calculating the translational vibration in that direction. This model allows for the consideration of the support stiffness of the bearings supporting the gear shafts.

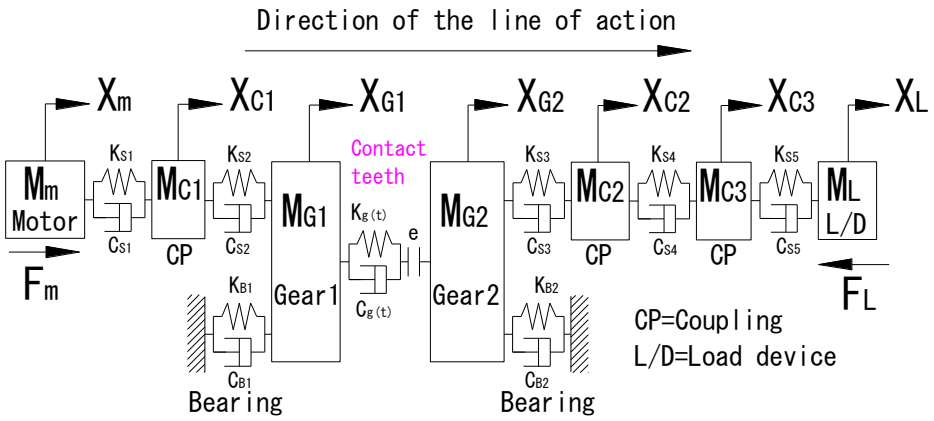

Figure 3: Seven-degree-of-freedom (7-DOF) Translational vibration analysis model

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$X_M, X_{C1}, X_{G1}, X_{G2}, X_{C2}, X_{C3}, X_L$：Motor, Coupling 1, Small gear, Large gear, Coupling 2, Coupling 3, and Linear displacement obtained by converting the torsional angular displacement of the powder brake to the line of action of the gear

$M_M, M_{C1}, M_{G1}, M_{G2}, M_{C2}, M_{C3}, M_L$：Motor, Coupling 1, Small gear, Large gear, Coupling 2, Coupling 3, and Equivalent mass obtained by converting the inertia of the powder brake to the line of action of the gear

$K_{S1}, K_{S2}, K_{S3}, K_{S4}, K_{S5}$：Motor shaft, Gear input shaft, Gear output shaft, Torque Converter Shaft, and Equivalent stiffness of the powder brake shaft, calculated by converting its torsional stiffness to the line of action of the gear

$C_{S1}, C_{S2}, C_{S3}, C_{S4}, C_{S5}$：Motor shaft, Gear input shaft, Gear output shaft, Torque Converter Shaft, Torque Converter Shaft, and Equivalent damping coefficient obtained by converting the torsional damping coefficient of the powder brake shaft to the line of action of the gear

K_{B1}, K_{B2}：Radial support stiffness of the bearings supporting the small gear shaft and the large gear shaft

C_{B1}, C_{B2}：Damping coefficient of the bearings supporting the small gear shaft and the large gear shaft

$K_{g1(t)}, K_{g2(t)}$：The bite spring constant for the first and second pairs of teeth, $K_{g(t)}$, is the composite bite spring constant for the teeth.

$C_{g1(t)}, C_{g2(t)}$：The bite damping coefficients, $C_{g(t)}$, for the first and second pairs of teeth are the composite damping coefficients of the teeth.

F_m, F_L：Equivalent load obtained by converting the torque acting on the motor and load device to the line of action of the gear

Figure 4 shows a 7-degree-of-freedom (7-DOF) torsional vibration analysis model of a power-absorbing gear tester. This model allows for the direct analysis of torsional vibrations in the rotational direction of the gear train.

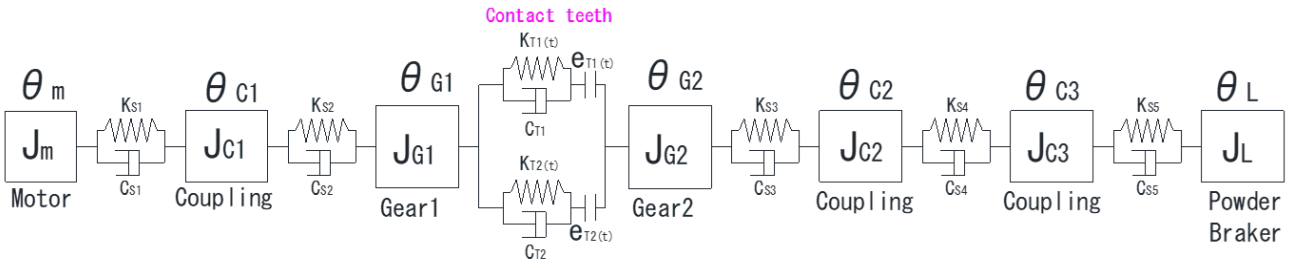

Figure 4: Seven-degree-of-freedom (7-DOF) torsional vibration analysis model

- $J_m, J_{C1}, J_{G1}, J_{G2}, J_{C2}, J_{C3}, J_L$：Motor, coupling C1, Small gear, Large gear, Coupling C2, Coupling C3, and Inertia (moment of inertia) of a powder brake
- $K_{S1}, K_{S2}, K_{S3}, K_{S4}, K_{S5}$：Motor shaft, Gear input shaft, Gear output shaft, Torque converter shaft, and Torsional Rigidity of the Powder Brake Shaft
- $C_{S1}, C_{S2}, C_{S3}, C_{S4}, C_{S5}$：Motor shaft, Gear input shaft, Gear output shaft, Torque converter shaft, and Torsional Damping Coefficient of the Powder Brake Shaft

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- $\theta_m, \theta_{C1}, \theta_{G1}, \theta_{G2}, \theta_{C2}, \theta_{C3}, \theta_L$: Motor, Coupling C1, Small gear, Large gear, Coupling C2, Coupling C3, and Angular Displacement of the Powder Brake
- $K_{T1(t)}, K_{T2(t)}$: Spring Constant of the Teeth in the First and Second Pairs of a Set of Gears
- C_{T1}, C_{T2} : Damping coefficient between the teeth of the first and second sets of gears in a pair of gears
- $e_{T1(t)}, e_{T2(t)}$: The clearance between the teeth of the first and second sets of gears

4． Vibration Analysis Methods and Analysis Software

Using the two models shown in Figures 3 and 4, we performed vibration analyses on a power-absorbing gear testing machine and compare the results below.

The gear vibration analysis method and software were developed by the author through many years of research. For details, please refer to References [1–2]. Since it is necessary to analyze the meshing stiffness of a pair of gears and the support stiffness of the bearings prior to the analysis, the author used specialized three-dimensional finite element method software developed by the author for these analyses as well [3–5].

5． Vibration Analysis Results

Figure 5 shows the Campbell diagram of the vibration acceleration of the gear analyzed using the model shown in Figure 3, and Figure 6 shows the Campbell diagram of the vibration acceleration of the gear analyzed using the model shown in Figure 4. A comparison of Figure 5 and Figure 6 reveals that the results are nearly identical.

Figure 7 shows the Campbell plot of the measured vibration acceleration of the gear. Comparing Figure 7 with Figures 5 and 6, it can be seen that the resonance frequencies are nearly the same, the resonance frequency peaks in the measured data are significantly larger than the calculated values. This is believed to be due to the fact that the vibration analysis did not account for gear manufacturing errors or assembly accuracy, which resulted in lower vibration values.

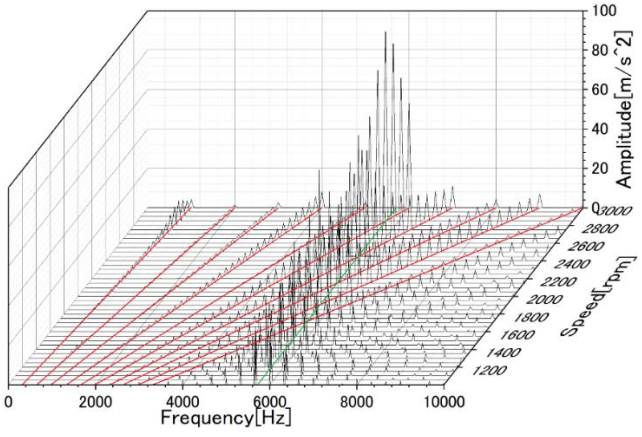

Figure 5: Campbell plot of the vibration acceleration of the gear

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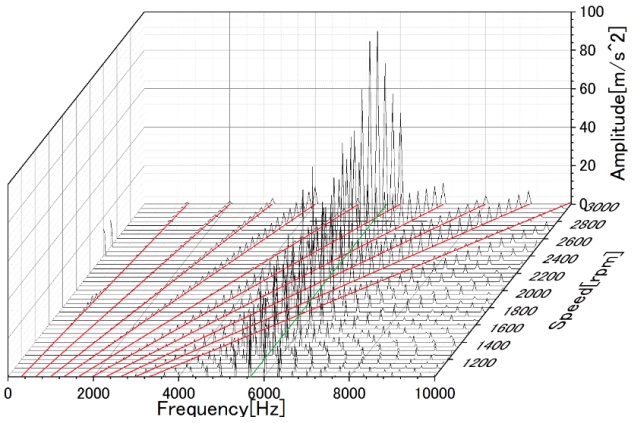

Figure 6: Campbell plot of the vibration acceleration of the gear

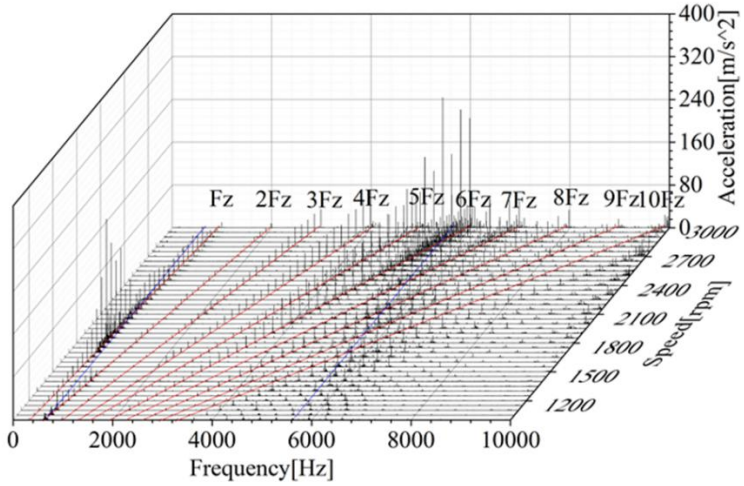

Figure 7: Campbell plot of vibration acceleration measurements for the large gear

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