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2023 年 8 月 22 日　Email: shutingnpu@yahoo.co.jp

Vibration Analysis of a Pair of Spur Gears Using a Power Circulation Gear Test Equipment

By Prof. Shuting Li

1． Abstract

To reduce gear vibration and noise, it is necessary to be able to accurately analyze gear vibration theoretically. However, it is well known that gear vibration analysis is not straightforward, as gear vibration is influenced by numerous factors—including gear machining accuracy, assembly accuracy, tooth surface finishing, and tooth surface friction—as well as the number of meshing teeth and the meshing position. I am one of many researchers who have conducted research on gear vibration analysis. Since around 1987, I have been exploring methods for analyzing gear vibration through my own research and have also been involved in the development of specialized software; however, even after more than 20 years, my research had not progressed significantly. In recent years, there have been major advances in gear vibration analysis, so I would like to briefly introduce the results of that analysis here.

2． Power-Circulation-Type Gear Testing Machine

To measure the vibration of a pair of spur gears, a power-circulating gear tester, as shown in Figure 1, can be used. Since power-circulating gear testers can apply larger loads than power-absorbing gear testers, this type of equipment is widely used for gear vibration measurements. Figures 1 and 2 show a power-circulating gear tester designed and built by the author.

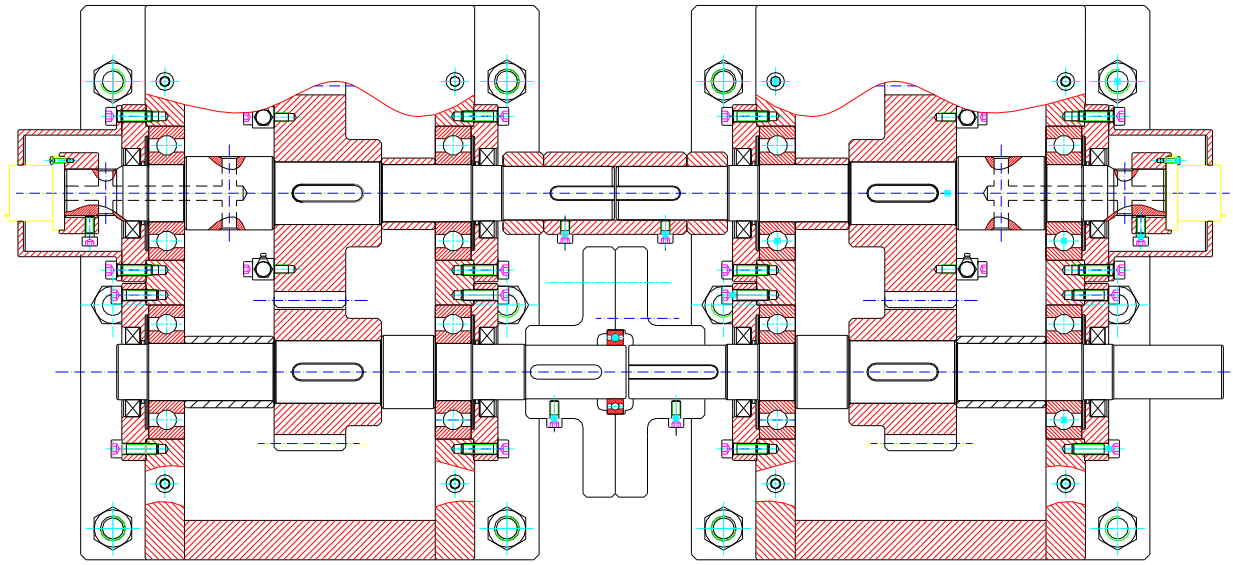

Figure 1: Cross-sectional view of the structure of a power-cycle gear testing machine

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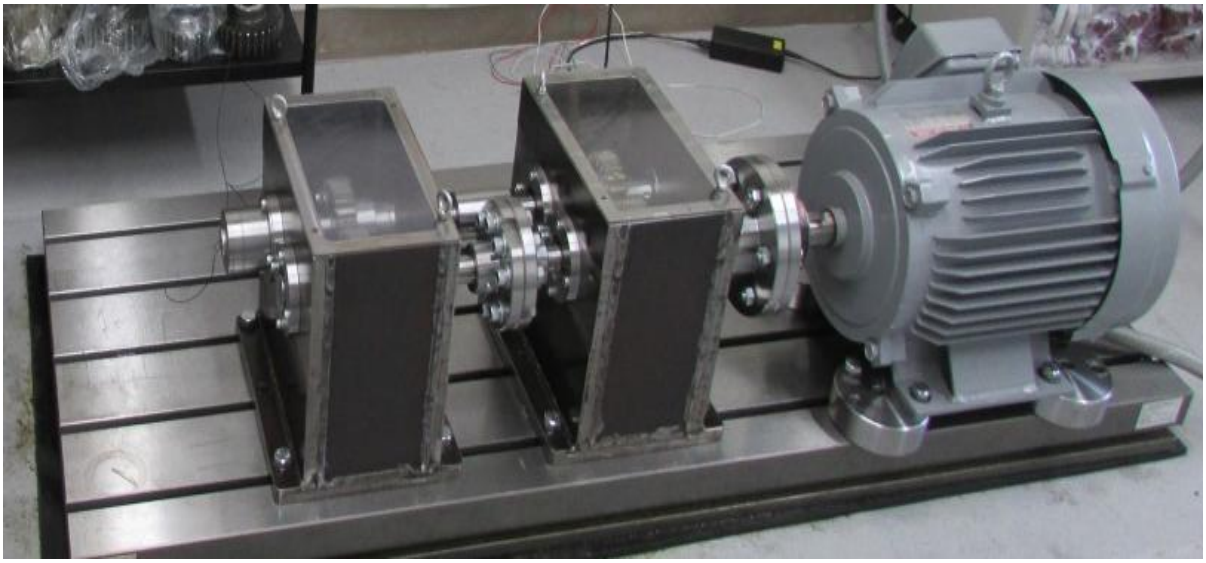

Figure 2: Cross-sectional view of the structure of a power-cycle gear testing machine

3． Gear Vibration Analysis Modeling

To analyze torsional vibrations in the rotational direction of a pair of spur gears, we developed vibration analysis models for the power-circulating gear tester shown in Figures 1 and 2. Figure 3 shows the proposed 7-degree-of-freedom (7-DOF) vibration analysis model, and Figure 4 shows the proposed 8-degree-of-freedom (8-DOF) vibration analysis model. We performed vibration analyses of the power-circulating gear tester using these models and compared the results.

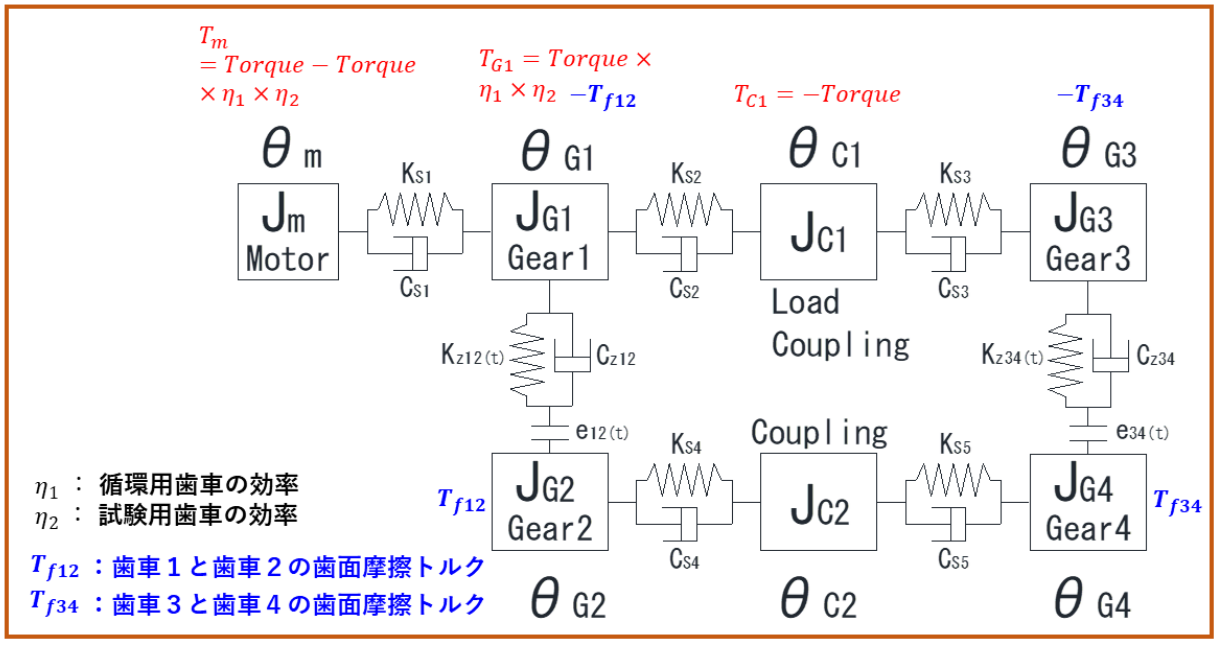

Figure 3: Seven-degree-of-freedom (7-DOF) model for vibration analysis

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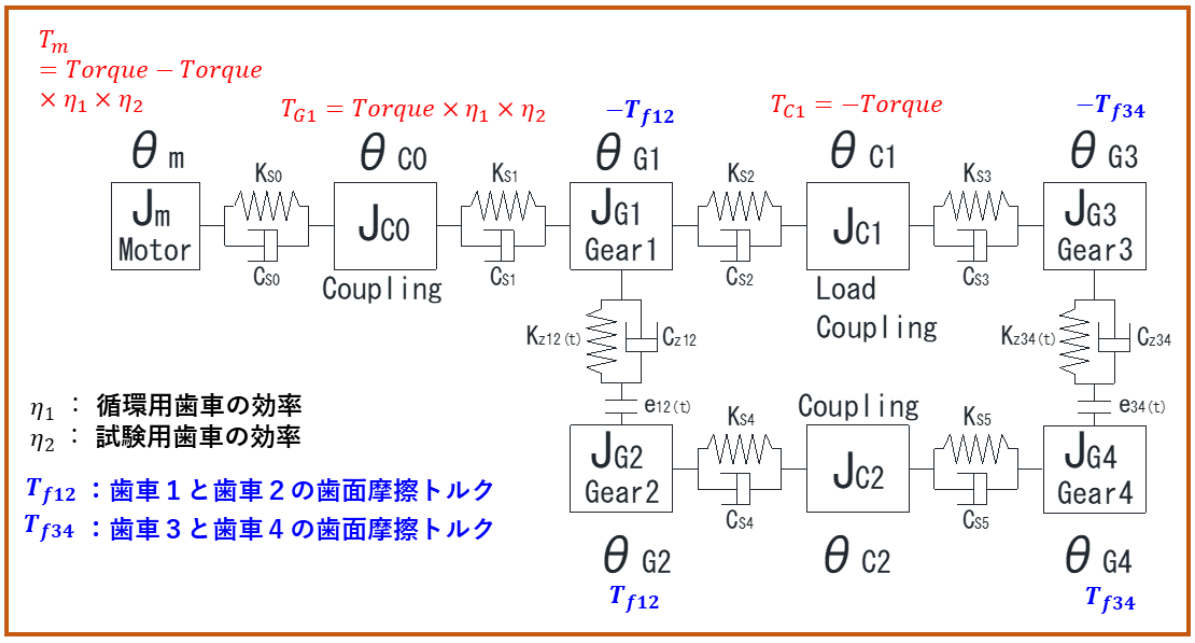

Figure 4: Eight-degree-of-freedom (8-DOF) model for vibration analysis

- $\theta_m, \theta_{C0}, \theta_{G1}, \theta_{G2}, \theta_{G3}, \theta_{G4}, \theta_{C1}, \theta_{C2}, \theta_{C3}$: Motor, Coupling C0, Small Gear for Power Circulation, Large Gear for Power Circulation, Small Gear for Vibration Testing, Large Gear for Vibration Testing, Load Coupling C1, Torsional Angular Deformation of Cylindrical Joint C2
- $J_m, J_{C0}, J_{G1}, J_{G2}, J_{G3}, J_{G4}, J_{C1}, J_{C2}, J_{C3}$: Motor, Coupling C0, Small gear for the power cycle, Large gear for the power cycle, Small gear for vibration testing, Large gear for vibration testing, Load Coupler C1, Moment of Inertia of Cylindrical Joint C2
- $K_{S0}, K_{S1}, K_{S2}, K_{S3}, K_{S4}, K_{S5}$: The shaft between the motor and coupling C0, The shaft between coupling C0 and the small gear for the power circuit, The shaft between the small gear for the power circuit and load coupling C1, The shaft between load coupling C1 and the small gear used for vibration testing, Shaft between the large gear for the power circuit and the C2 cylindrical coupling, Torsional stiffness of the shaft between the large gear and the C2 cylindrical coupling used in vibration testing
- $C_{S0}, C_{S1}, C_{S2}, C_{S3}, C_{S4}, C_{S5}$: Torsional damping coefficients for the axes corresponding to $K_{S0}, K_{S1}, K_{S2}, K_{S3}, K_{S4}, K_{S5}$
- $K_{Z12(t)}, K_{Z34(t)}$ ：For power circulation, Total meshing stiffness of a pair of gears for vibration testing
- $C_{Z12(t)}, C_{Z34(t)}$ ：For power circulation, Total damping coefficient of a pair of meshing gears for vibration testing
- $e_{12(t)}, e_{34(t)}$ ：For power circulation, Backlash in Gears for Vibration Testing and Tooth Profile Errors in a Pair of Gears

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4． Vibration Analysis Methods and Analysis Software

The gear vibration analysis method and software were developed by the author through many years of research. For details, please refer to References [1–2]. Since it is necessary to analyze the meshing stiffness of a pair of gears and the support stiffness of the bearings prior to the analysis, the author used specialized 3D finite element method software developed by the author for these analyses as well [3–5].

5． Vibration Analysis Results

Figure 5 shows the Campbell plot of the vibration acceleration of Gear 4 analyzed using the 7-DOF model, and Figure 6 shows the Campbell plot of the vibration acceleration of Gear 4 analyzed using the 8-DOF model. Comparing Figure 6 with Figure 5, it can be seen that the peak of the first-order frequency component is slightly larger.

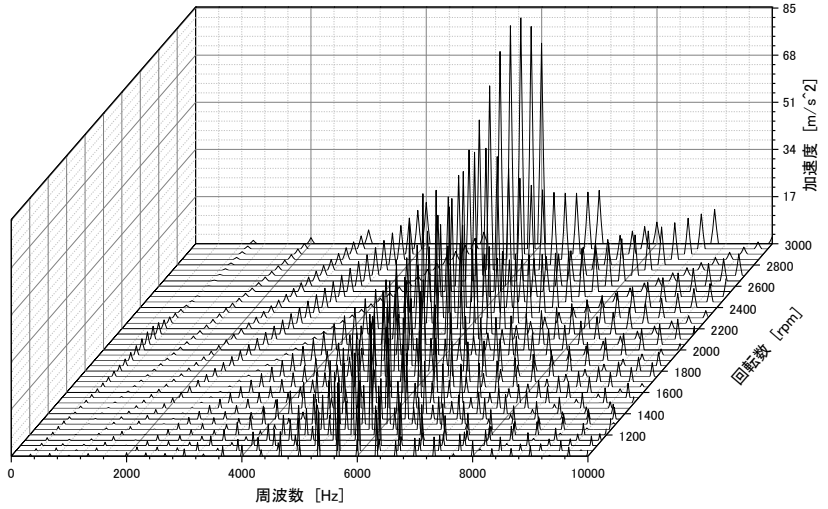

Figure 5: Campbell diagram of the vibration acceleration of Gear 4, analyzed using a 7-DOF model

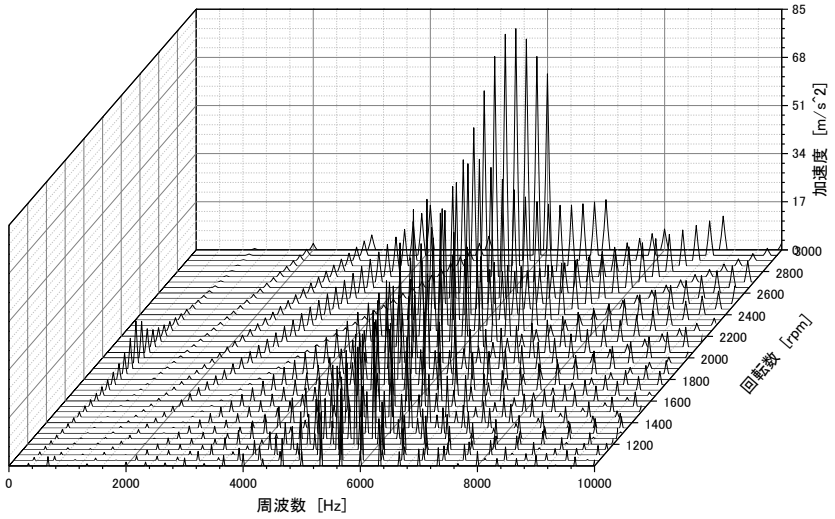

Figure 6: Campbell diagram of the vibration acceleration of Gear 4, analyzed using an 8-DOF model

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Figure 7 shows the Campbell diagram of the measured vibration acceleration of Gear 4. Comparing Figure 7 with Figure 8, it can be seen that while the resonance frequencies are nearly the same, the peak resonance frequency in the measured data is significantly higher than the calculated value. This is believed to be because the calculated values did not account for gear machining accuracy or assembly accuracy, which resulted in a lower peak in the calculated values. In the future, it will be necessary to perform vibration analysis that takes into account gear machining errors and assembly accuracy.

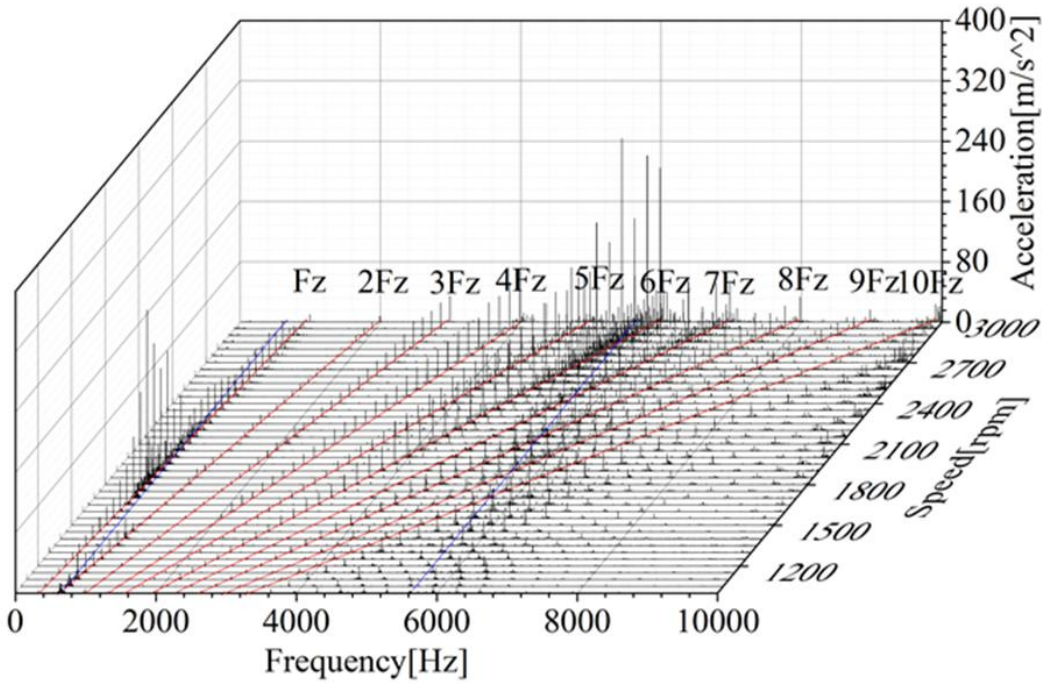

Figure 7: Campbell diagram of the measured vibration acceleration of Gear 4

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