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Meshing Stiffness Analyses of a Pair of Internal and External Spur Gears

By Prof. Shuting Li

1. Abstract

When analyzing gear vibration, it is necessary to determine the meshing stiffness of the teeth; however, since meshing stiffness varies depending on the meshing position of the teeth and is highly susceptible to manufacturing errors, assembly errors, and tooth surface finishing, actually determining meshing stiffness is a very difficult task. To determine meshing stiffness, the author spent a considerable amount of time developing specialized 3D finite element method (FEM) software. The following presents the results of analyses performed using this specialized software on the meshing stiffness of a pair of external spur gears, as well as the meshing stiffness when an external spur gear meshes with an internal spur gear. This software can analyze meshing stiffness even when the gears have assembly errors, manufacturing errors, and tooth surface finishing [1–3].

2. Meshing Stiffness Analysis of External Spur Gears

Figure 1 shows the 3D, FEM model used to analyze the contact problem of a pair of external spur gears. To determine the meshing stiffness of the gears using this model, one meshing cycle (pitch) was divided into 12 meshing positions, and the tooth meshing stiffness was analyzed at each position by performing contact analysis of the gear pair using the three-dimensional finite element method. During the analysis, a torque of 400 Nm was applied to the gear. Table 1 shows the tooth meshing stiffness at each meshing position as determined by the finite element method analysis.

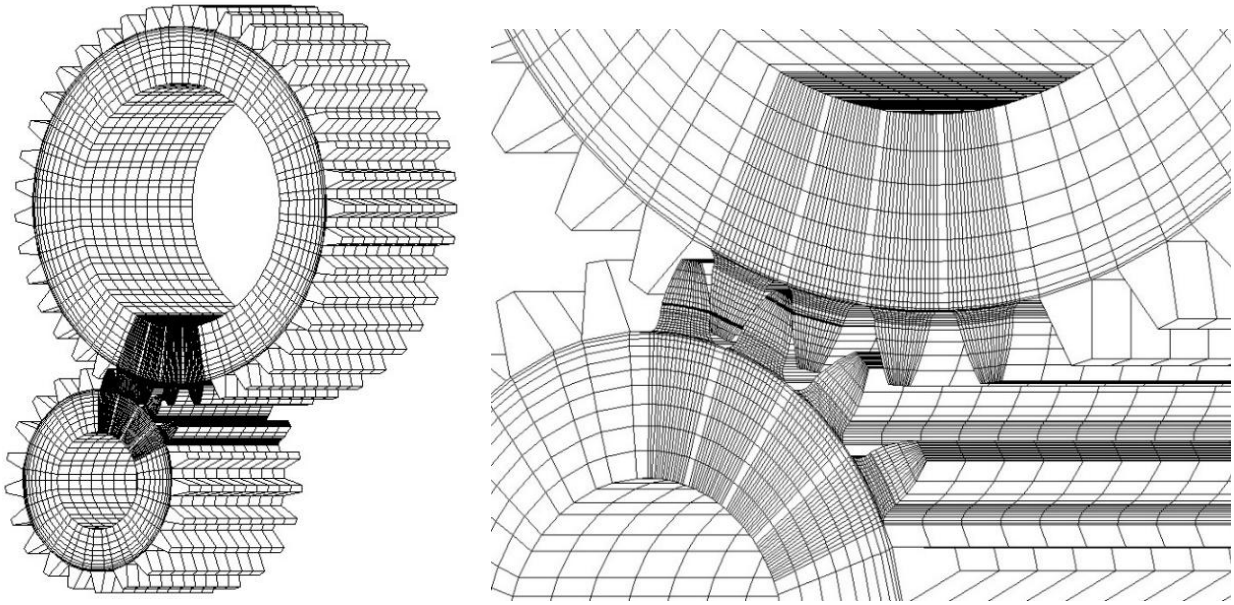

Figure 1: Finite element method model for contact analysis of a pair of cylindrical gears

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Table 1: Tooth engagement stiffness when two eternal gears mesh [Unit: N/m]

Meshing position	Mesh stiffness of the first pair of teeth	Mesh stiffness of the second pair of teeth	Total Stiffness
1	3.3560×10^8	5.3099×10^8	8.6659×10^8
2	3.9510×10^8	5.0978×10^8	9.0488×10^8
3	4.2587×10^8	4.8848×10^8	9.1435×10^{98}
4	4.5542×10^8	4.6488×10^8	9.2031×10^8
5	4.7728×10^8	4.3419×10^8	9.1146×10^8
6	5.0227×10^8	3.9774×10^8	9.0001×10^8
7	5.2635×10^8	3.4862×10^8	8.7497×10^8
8	5.8713×10^8		5.8713×10^8
9	5.9566×10^8		5.9566×10^8
10	6.0054×10^8		6.0054×10^8
11	5.9731×10^8		5.9731×10^8
12	5.7732×10^8		5.7732×10^8

3． Meshing Stiffness Analysis of External Spur Gears

Figure 2 shows the 3D, FEM model used to analyze contact problem when an external spur gear meshes with an internal spur gear. To determine the meshing stiffness of the gears using this model, one meshing cycle (pitch) was divided into 10 meshing positions, and the tooth meshing stiffness was analyzed at each position by performing contact analysis of the gear pair using the 3D, FEM. During the analysis, a torque of 500 Nm was applied to the internal gear. Table 2 shows the meshing stiffness at each meshing position as determined by the finite element analyses.

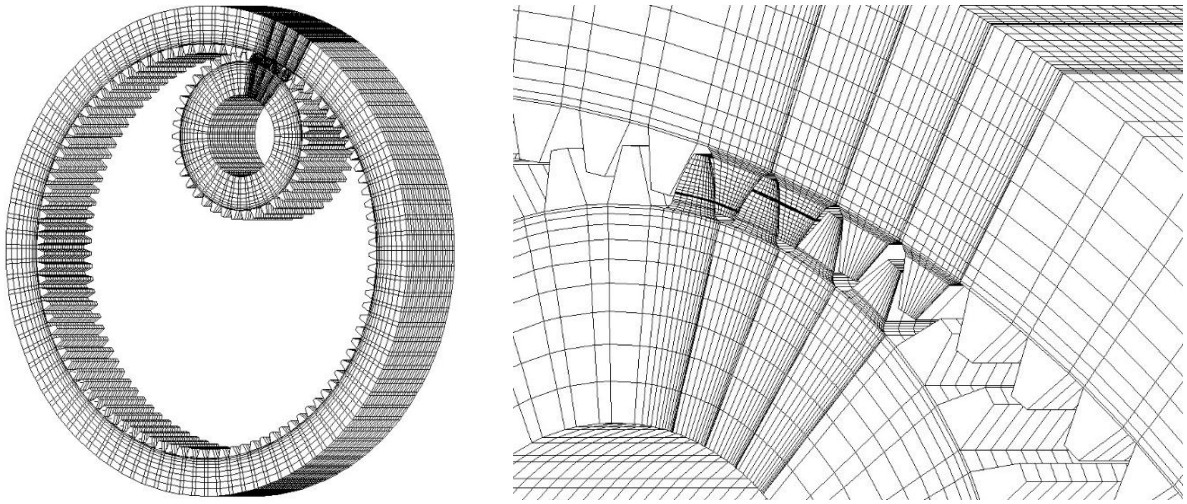

Figure 2: FEM model for analyzing mesh stiffness of an external spur gear meshes with an internal spur gear

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Table 2: Meshing stiffness of an external gear with an internal gear [Unit: N/m]

Meshing position	Mesh stiffness of the first pair of teeth	Mesh stiffness of the second pair of teeth	Total Stiffness
1	4.0019×10^{8}	6.3442×10^{8}	1.0346×10^{9}
2	4.3585×10^{8}	6.3165×10^{8}	1.0675×10^{9}
3	4.7516×10^{8}	6.1111×10^{8}	1.0863×10^{9}
4	5.1004×10^{8}	5.7914×10^{8}	1.0892×10^{9}
5	5.4033×10^{8}	5.4534×10^{8}	1.0857×10^{9}
6	5.7509×10^{8}	5.0064×10^{8}	1.0756×10^{9}
7	6.0654×10^{8}	4.4893×10^{8}	1.0555×10^{9}
8	6.3421×10^{8}	3.8809×10^{8}	1.0223×10^{9}
9	7.1836×10^{8}		7.1836×10^{8}
10	7.1406×10^{8}		7.1406×10^{8}

Using the results in Tables 1 and 2, the curves of tooth meshing stiffness calculated using the interpolation method are shown in Figures 3 and 4, respectively. The horizontal axis of these figures represents the angle indicating the rotational position of the gear, and the vertical axis represents the interpolated tooth meshing stiffness.

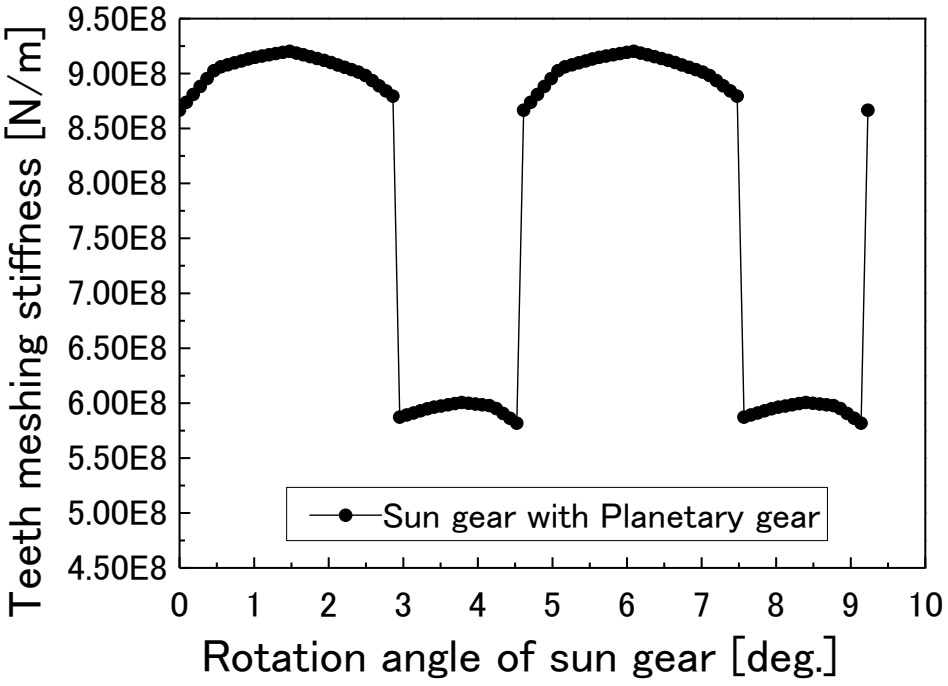

Figure 3: Tooth meshing stiffness when two external gears mesh

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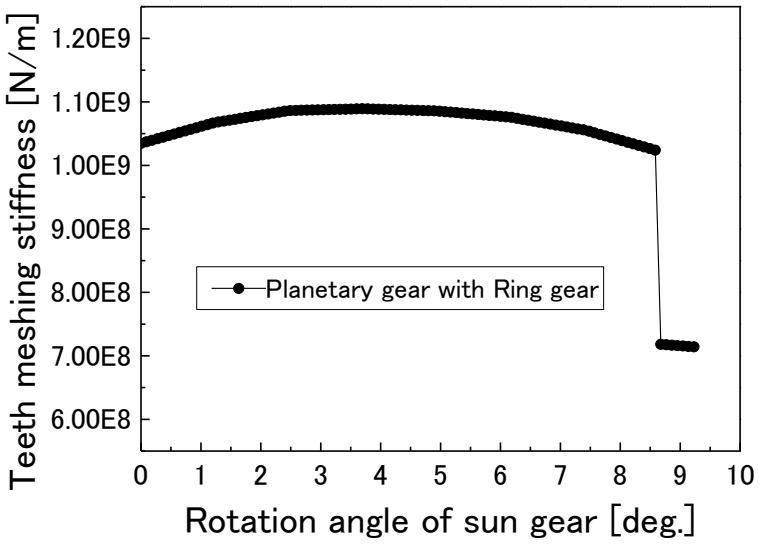

Figure 4: Tooth meshing stiffness when an external gear and an internal gear mesh

References：

(1) Shuting Li, “Effects of Machining Errors, Assembly Errors and Tooth Modifications on Load-Carrying Capacity, Load-Sharing Rate and Transmission Error of a Pair of Spur Gear”, Mech. Mach. Theory, Vol. 42, Issue 6, 2007, pp.698-726.

(2) Shuting Li, “Finite Element Analyses for Contact Strength and Bending Strength of a Pair of Spur Gear with Machining Errors, Assembly Errors and Tooth Modifications”, Mech. Mach. Theory, Vol. 42, Issue 1, 2007, pp.88-114.

(3) Shuting Li, “Gear Contact Model and Loaded Tooth Contact Analysis of a Three-Dimensional, Thin-Rimmed Gear”, Trans. ASME, J. Mech. Des., Vol. 124, Issue 3, Sept. 2002, pp.511-517.