文責：島根大学総合理工学部機械設計研究室教授　李　樹庭
2023 年 8 月 22 日 Email: shutingnpu@yahoo.co.jp

FEM for Design and Strength Analysis of Strain Wave Gearing

By Prof. Shuting Li

1． Abstract

Strain wave gearings are characterized by their high reduction ratios, compact size, and light weight, making them widely used in various fields, including industrial robots. However, because these mechanisms employ ultra-thin-walled spur gears and rely on the elastic deformation of these gears to achieve tooth meshing, the meshing motion of this type of reducer becomes extremely complex, which in turn makes its design and strength analysis extremely challenging. The author began research on the design and strength analysis of wave gear mechanisms in 1986. Using a dedicated finite element method and software developed over a long period of time, the author has made it possible to perform contact analysis on wave gear mechanisms; the results are briefly presented here. For details on this research, please refer to References (1-2)，

2． Deep-Groove Ball Bearings Under Study

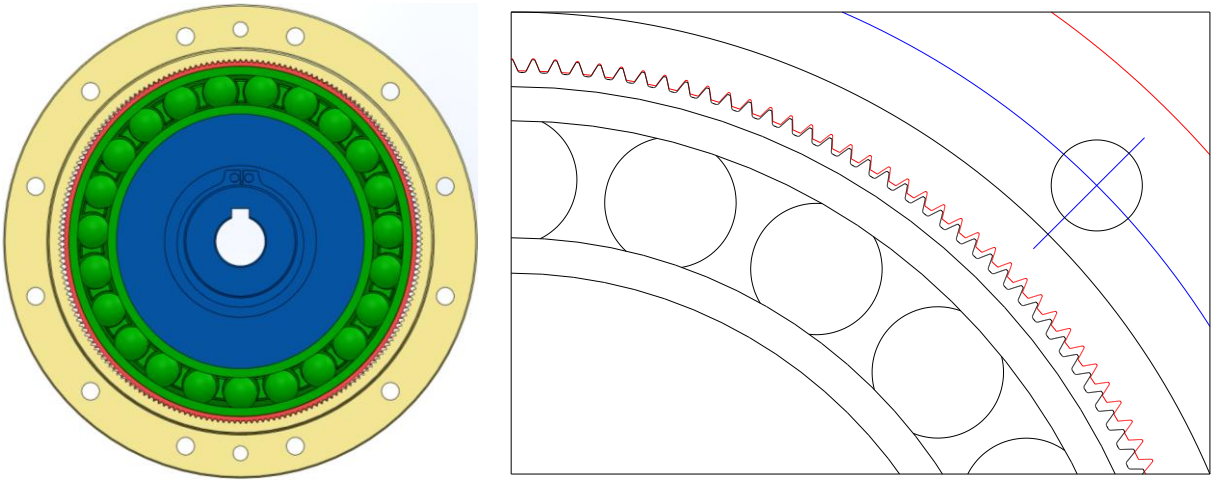
Figure 1 shows the input and output screens of the design calculation software we developed for wave gear mechanisms. Since this software is integrated into AutoCAD, it can be used as a feature of AutoCAD. As shown in Figure 1(a), once the specifications for F/S, C/S, and W/G are entered into the “Input” field and the “Calculate” command is executed, the wave gear mechanism is immediately designed, as shown in Figure 1(b) on the right. This software offers a wide range of tooth profiles (such as involute curves and modified involute curves) as optional features, allowing users to select the appropriate profile while designing the wave gear mechanism.

(a)　Design calculation software　　(b)　Calculation results

Figure 1: Development of design calculation software for wave gear mechanisms

文責：島根大学総合理工学部機械設計研究室教授　李　樹庭
2023 年 8 月 22 日 Email: shutingnpu@yahoo.co.jp

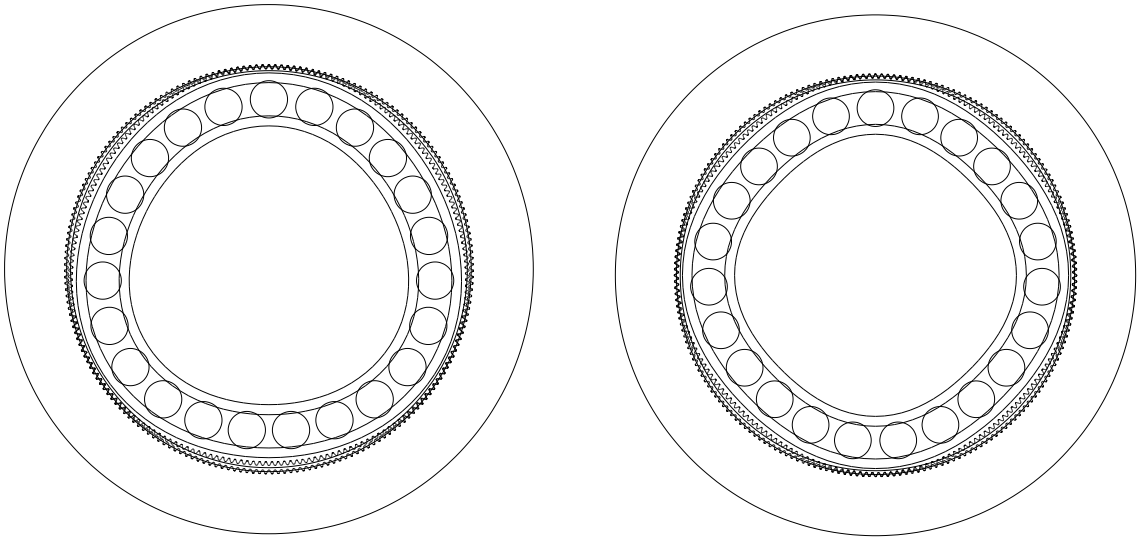
Figure 2(a) shows a cross-sectional view of the opening in the three-dimensional wave gear mechanism created in SolidWorks. Figure 2(b) is a partial enlargement of Figure 1(b).



(a) Results in SolidWorks　　(b) Enlarged view of the bite alignment

Figure 2: Enlarged view of the wave gear mechanism and gear meshing created in SolidWorks

We also developed design calculation software to design multi-lobe wave gear mechanisms. Figure 3(a) shows a three-lobe wave gear mechanism designed using the developed software, and Figure 4(a) shows a four-lobe wave gear mechanism designed using the same software. Since the W/G of a conventional wave gear mechanism is elliptical, it has two major axes, making this a two-lobe wave gear mechanism. As shown in Figures 3 and 4, the W/G of the three-lobe and four-lobe wave gear mechanisms have triangular and square shapes, respectively, and since they have three and four major axes, respectively, these mechanisms are referred to as three-lobe and four-lobe wave gear mechanisms.



(a) Three-Lobe wave gear mechanism　　(b) Four-lobe wave gear mechanism

Figure 3: Design of a multi-lobe wave gear mechanism

文責：島根大学総合理工学部機械設計研究室教授　李　樹庭
2023 年 8 月 22 日 Email: shutingnpu@yahoo.co.jp

3. Three-Dimensional Finite Element Method for Contact Strength Analysis of Wave Gear Mechanisms

Next, we present the results of strength calculations for the wave gear mechanism analyzed using specialized 3D finite element method software developed by the author. Strength analysis was performed using a top-hat-type wave gear mechanism. Figure 4 shows the element mesh model used for the FEM analysis.

Figure 5 shows the distribution of the maximum tooth-face pressure for each tooth in the range of (-23° to +38°) near the major axis. Figure 6 shows the distribution of the maximum root bending stress for each tooth in the range of (-23° to +38°) near the major axis. Figure 7 shows the stress distribution along the circle of intersection between the diaphragm and the body. In the case of a top-hat-type wave gear mechanism, the critical stress area is located near the circle of intersection between the diaphragm and the body; however, to eliminate this critical area, the author designed the diaphragm and the body as a single arc (3). This eliminated structural failure in the F/S.

Figure 8 shows the results for the load on the ball surface and the contact pressure of a thin-walled ball bearing. These results make it possible to determine whether flaking will occur in the bearing.

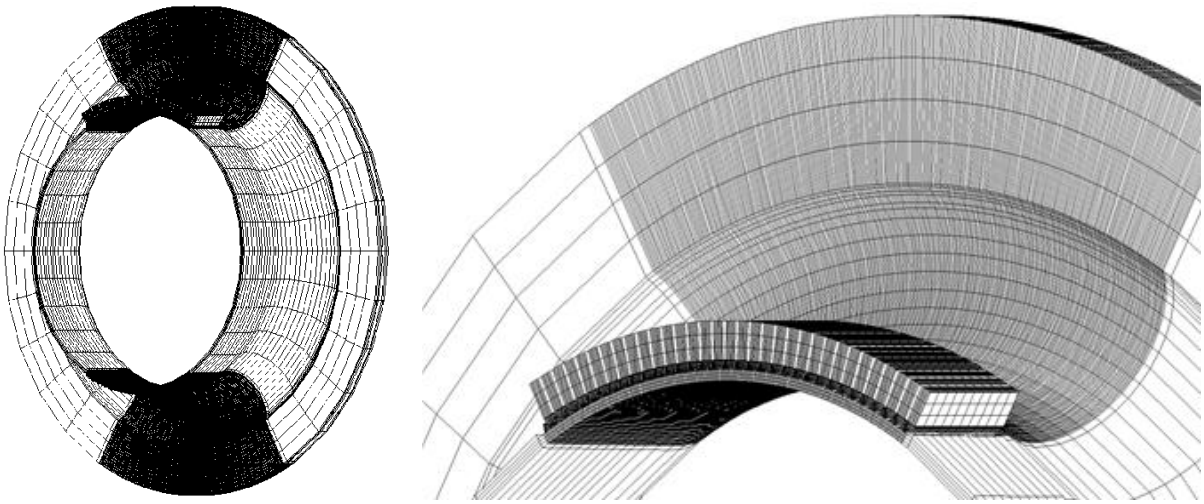


Figure 4: Top hat-type wave gear mechanism

文責：島根大学総合理工学部機械設計研究室教授　李　樹庭
2023 年 8 月 22 日 Email: shutingnpu@yahoo.co.jp

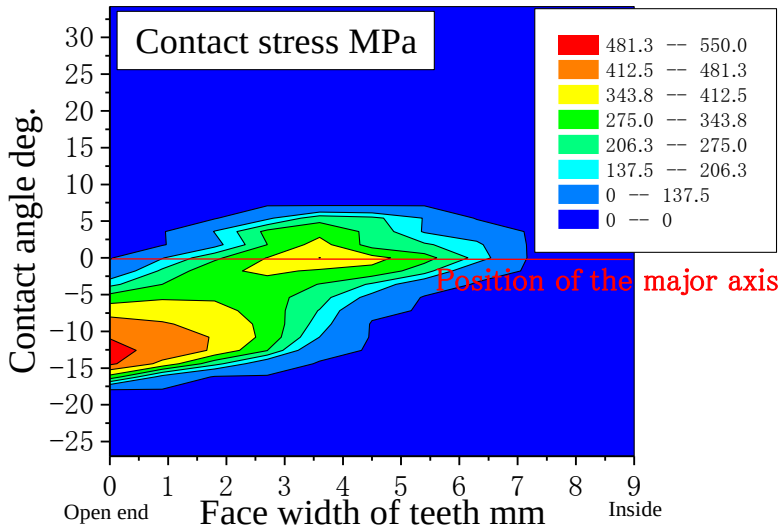


Figure 5: Contour map of surface pressure distribution on a tooth surface4

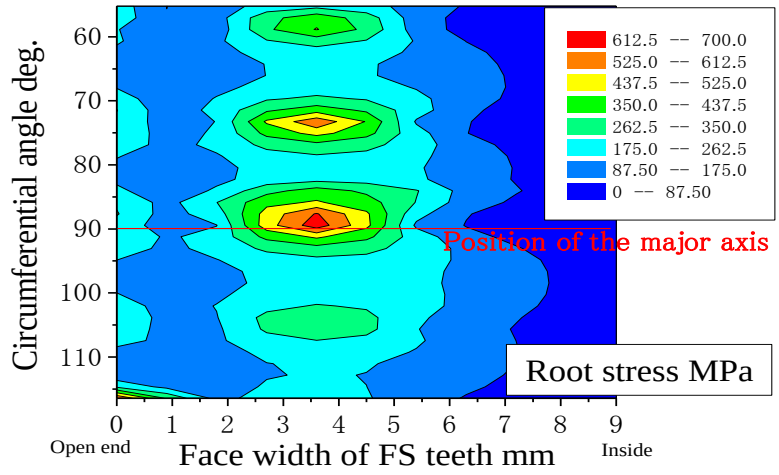


Figure 6: Contour map of bending stress distribution at the tooth root

文責：島根大学総合理工学部機械設計研究室教授　李　樹庭
2023 年 8 月 22 日 Email: shutingnpu@yahoo.co.jp

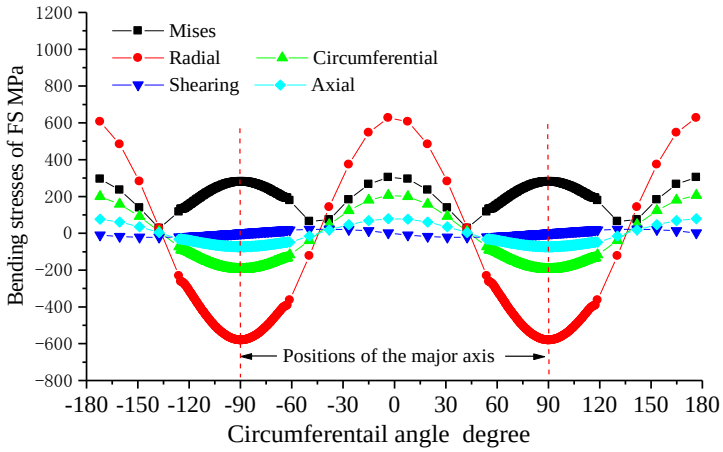


Figure 7: Stress distribution along the intersection circle between the diaphragm and the bore section

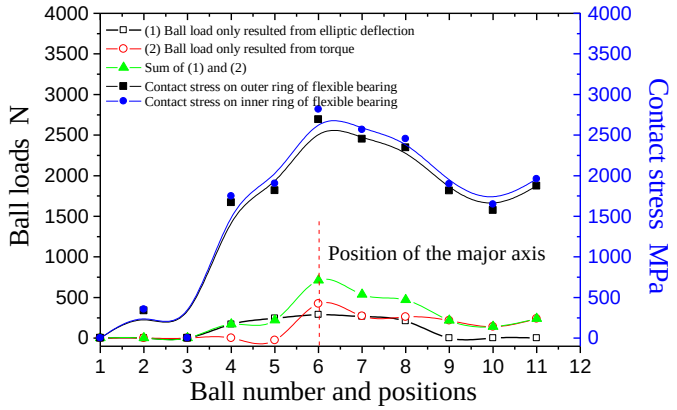


Figure 8: Load and contact pressure distribution on thin-walled bearing balls

4． Experimental Verification of the Analysis Results

We experimentally verified the validity of the FEM analysis results using a cup-type wave gear mechanism. Figure 9 shows the element-mesh model used for the FEM contact analysis. Figure 10 shows an enlarged cross-sectional view of the opening. As shown in Figure 11, strain gauges were attached near the intersection of the diaphragm and the Bosse circle. Using the experimental setup shown in Figure 12, the strain waveforms of the diaphragm were measured with three-axis strain gauges while rotating the wave gear mechanism under load. The measurement results are shown in Figure 13. The FEM analysis results are shown in Figure 14. A comparison of Figures 14 and 13 reveals that the analysis results agree well with the experimental results.

文責：島根大学総合理工学部機械設計研究室教授　李　樹庭
2023 年 8 月 22 日 Email: shutingnpu@yahoo.co.jp

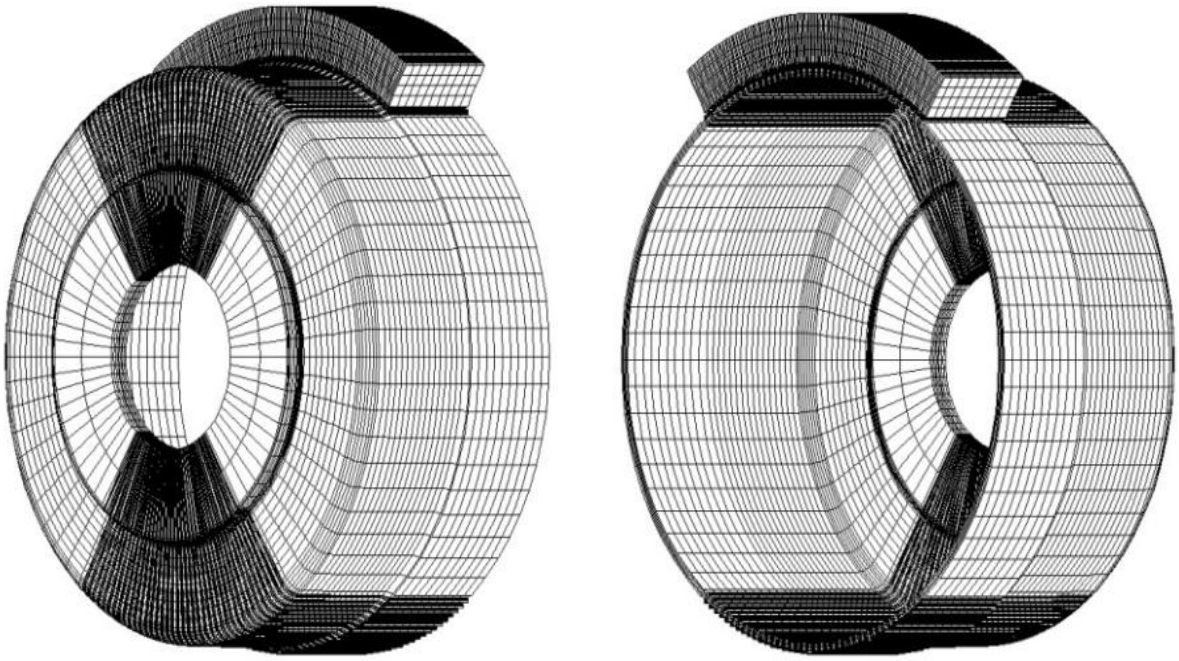
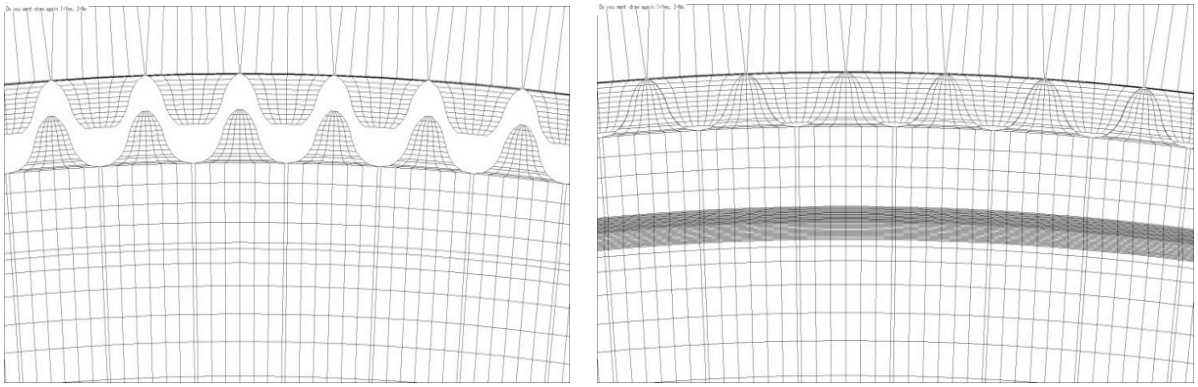


Figure 9: FEM element mesh model for contact analysis of a cup-type wave gear mechanism



(a) Before inserting the W/G　　(b) After inserting the W/G

Figure 10: Definitions of cross-sections 1 and 2

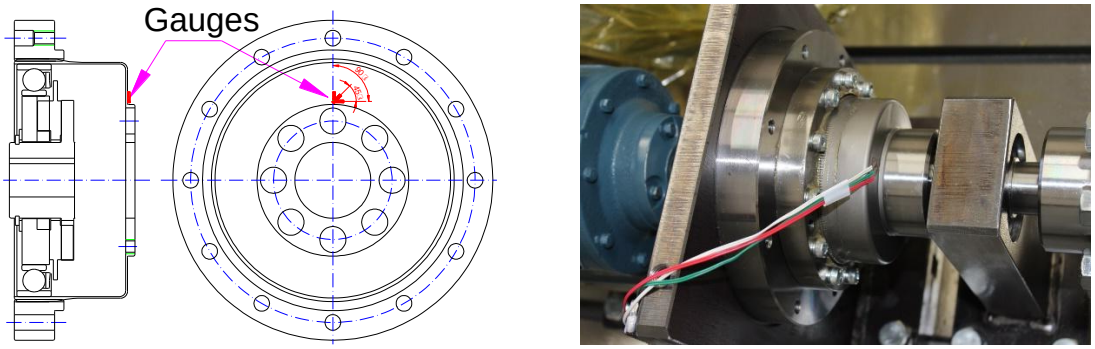


Figure 11: Strain gauge mounting locations

文責：島根大学総合理工学部機械設計研究室教授　李　樹庭
2023 年 8 月 22 日 Email: shutingnpu@yahoo.co.jp

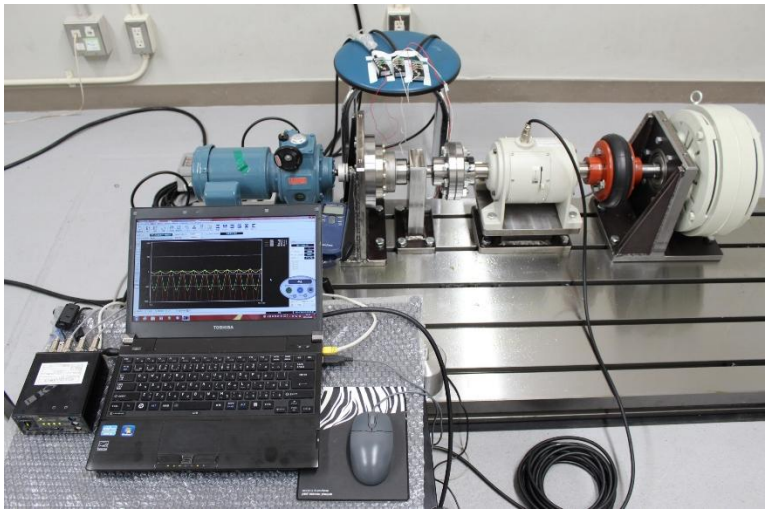


Figure 12: Experimental setup developed to measure dynamic strain

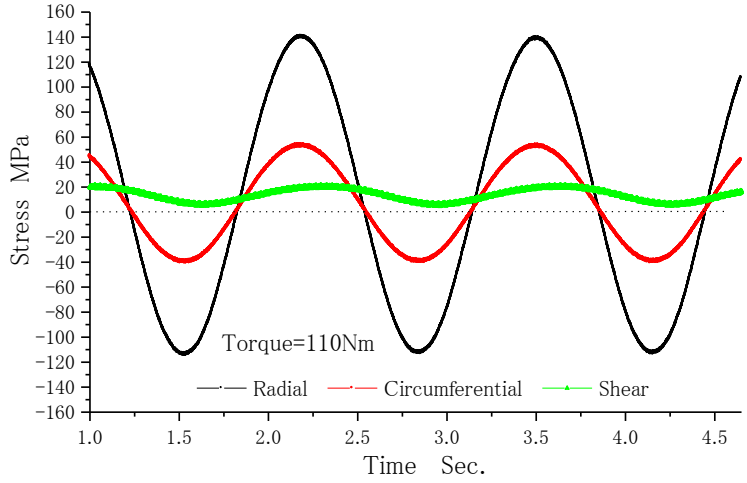


Figure 13: Measurement results

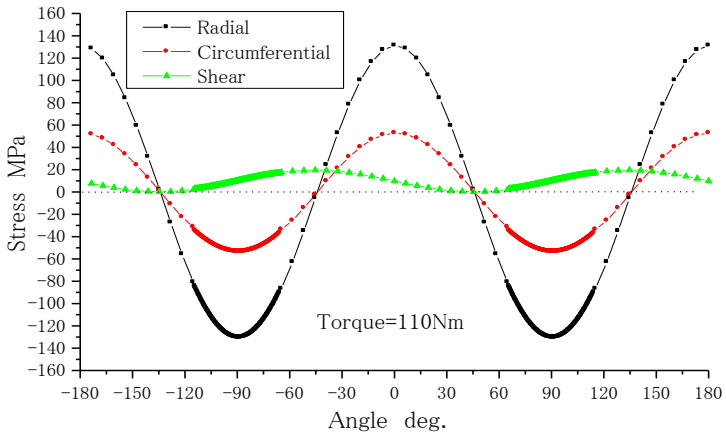


Figure 14: Analysis results from dedicated FEM software

文責：島根大学総合理工学部機械設計研究室教授　李　樹庭
2023 年 8 月 22 日 Email: shutingnpu@yahoo.co.jp

References：

(1) S. Li and Y. Shen, “Fatigue failure patterns and strength design methods of a harmonic drive device”, **The 11th World Congress in Mechanism and Machine Science**. Vol.2, pp.805-810, 2004

(2) S. Li, “Diaphragm stress analysis and fatigue strength evaluation of the flex-spline, a very thin-walled spur gear used in the strain wave gearing”, **Mechanism and Machine Theory**, Elsevier Press, Volume 104, 2016, pp.1-16

(3)　Li Shuting, New Wave Gear Device of the Top Hat Type, Japan Patent Office, JP 2007-16838 (January 25, 2007)