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Contact Strength Analysis of Three-Dimensional, Thin-Walled Spur Gears

By Prof. Shuting Li

1. Abstract

To reduce weight, the rim and web thicknesses of gears have been reduced. Consequently, when subjected to a load, thin-walled gears deform more easily than thick-walled gears. When calculating the strength of thick-walled gears, deformation of the rim and web is ignored, so gear strength can be roughly calculated using empirical formulas. However, when attempting to calculate the strength of thin-walled gears, it is necessary to account for structural deformation of the rim and web; consequently, determining the strength of thin-walled gears remains an unsolved challenge. To address this challenge, the author[1-4] developed a specialized three-dimensional (3D), finite element method (FEM) and corresponding software. Here, I will briefly present only the results of the research on the strength analysis of thin-walled gears; for details on this research, please refer to References[1-4].

2. Three-Dimensional Thin-Walled Spur Gears Under Study

As shown in Figure 1, this study focuses on a pair of spur gears with identical specifications. The number of teeth is Z = 50, the module is m = 4, and the shifting coefficients are x = 0. Figure 1(a) shows a thin-walled spur gear with a 3D structure, while Figure 1(b) shows a thick-walled gear used as the mating gear.

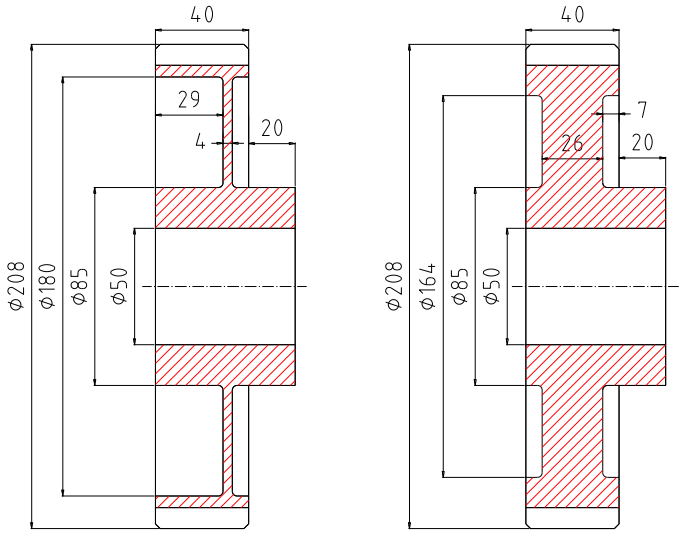


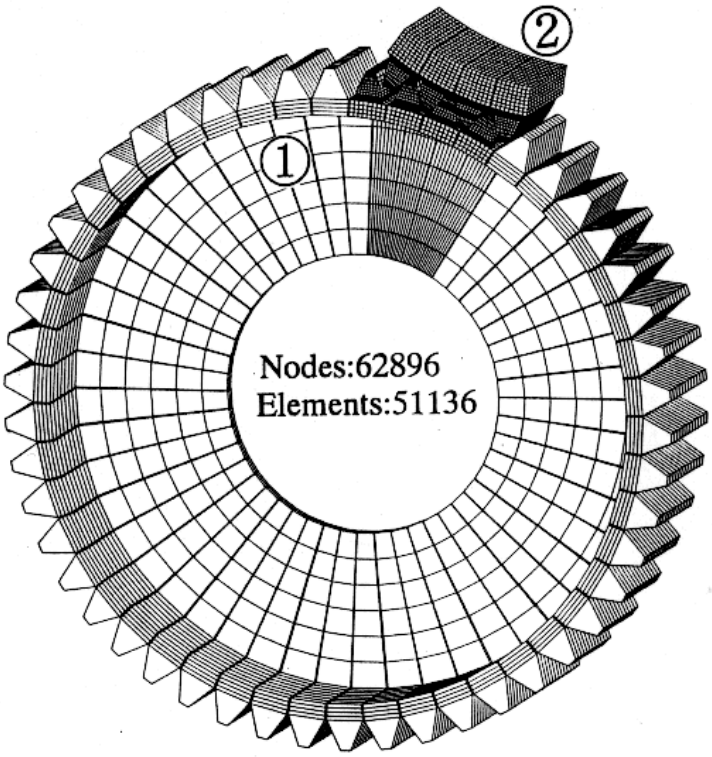
Figure 1: Thin-walled and thick-walled gears with three-dimensional structures under study

3. Contact Analysis of Thin-Walled Gears with Three-Dimensional Geometry

Figure 2 shows a 3D, FEM meshing model of a thin-walled gear meshing with a thick-walled gear. Using

this model, we analyzed the tooth-face contact strength and root bending strength of the thin-walled gear and investigated the effect of rim thickness on these strengths. Here, we briefly present only the analysis results; for details of the study, please refer to References[1–2].

Figure 3 illustrates the definition of the contact region on the tooth surface. As shown in Figure 3, the area hatched with red diagonal lines is the region used in the analysis of tooth contact problems and is referred to as the contact region. The X-axis represents the tooth root direction, and the Y-axis represents the tooth profile direction.



① Thin-walled gear； ② Thick-walled gear

Figure 2: FEM model for contact analysis of thin-walled spur gears

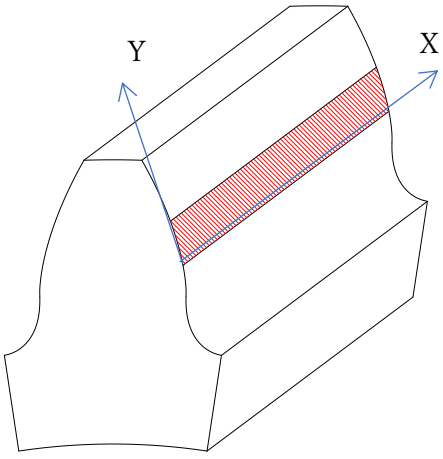


Figure 3: Definition of the contact area on the tooth surface (X-axis = tooth ridge; Y-axis = tooth profile)

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Figure 4 shows the load distribution over the contact area of a thin-walled gear, as analyzed using the developed 3D, FEM software. Figure 5 shows a contour plot of the tooth surface pressure distribution calculated based on the load distribution shown in Figure 4.

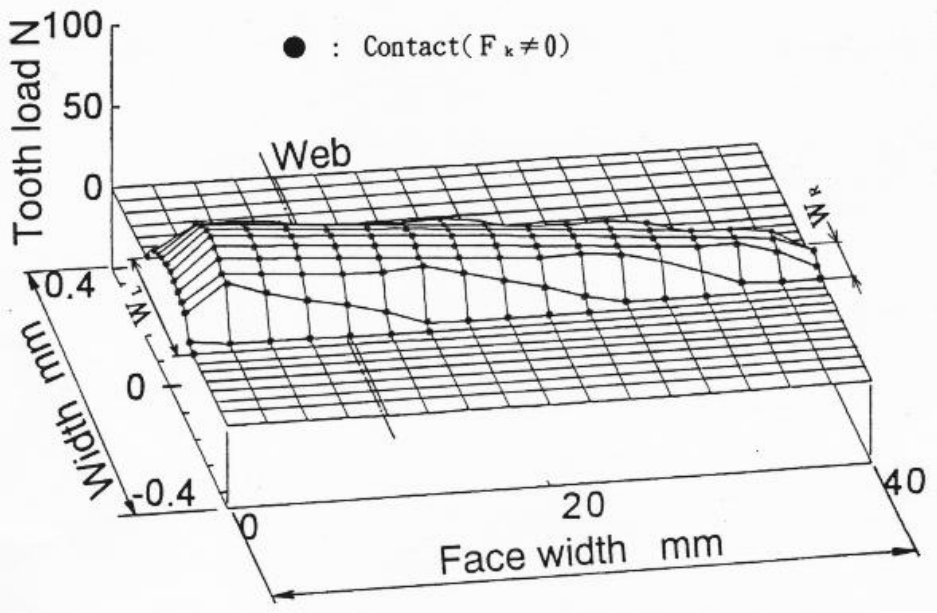

Figure 4: Distribution of tooth surface loads in the contact area shown in Figure 3

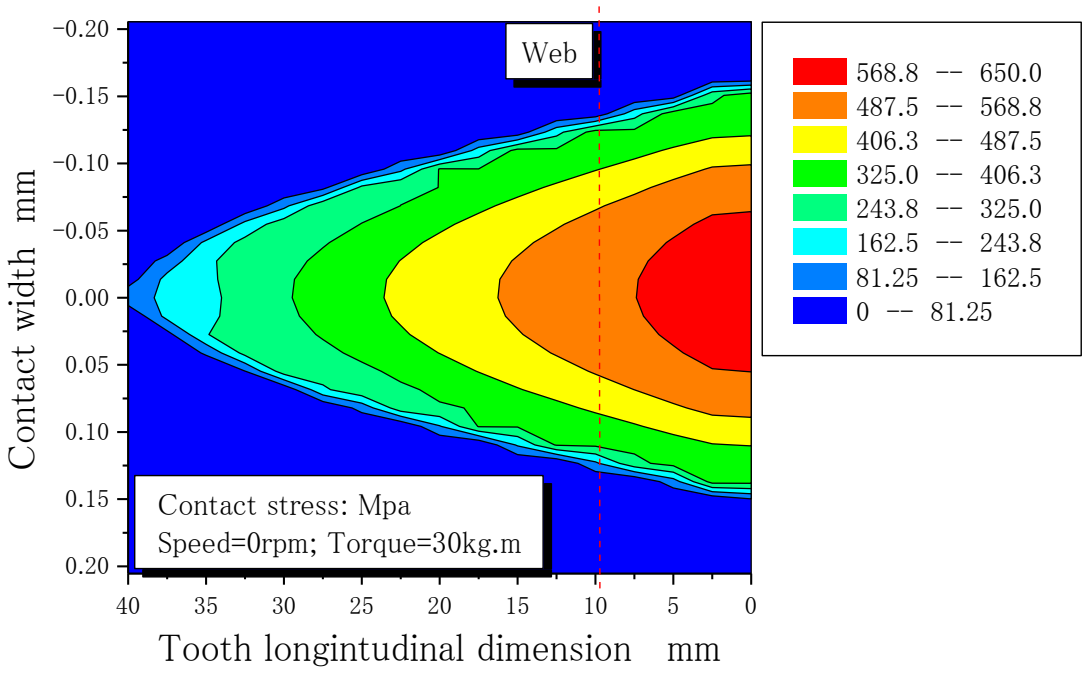

Figure 5: Distribution of contact stress (surface pressure) in the contact area

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Figure 6 shows the surface stress along the tooth profile of a thin-walled gear, as well as how this stress changes when the rim thickness varies.

Figure 7 shows the distribution of bending stress at the tooth root in the tooth root direction at the point of maximum stress. Figure 7 simultaneously shows the bending stresses on the tension and compression sides of the tooth root. The location of the web is also indicated by “Web.”

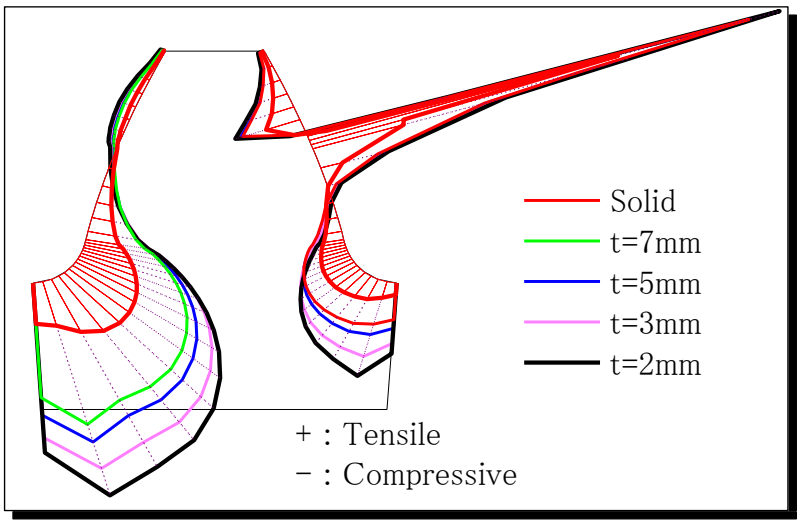

Figure 6: Surface stress along the tooth profile of a thin-walled gear

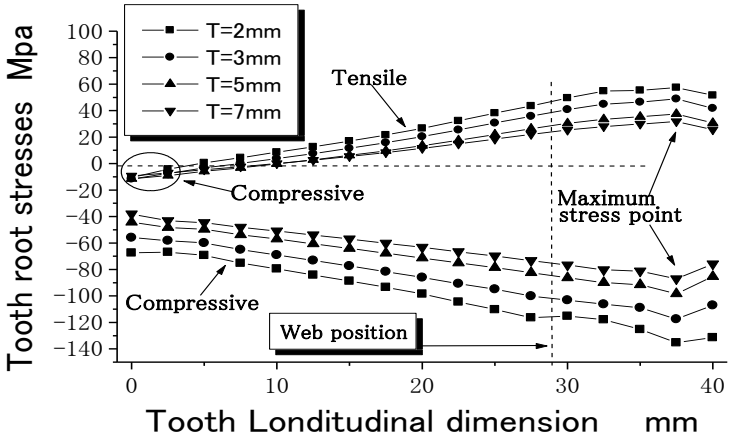

Figure 7: Root bending stress along the longitudinal direction at the point of maximum root stress

Figure 8 shows the stress distribution along the circle of intersection between the rim and the web. As shown in Figure 8, it can be seen that high stress occurs at approximately 94 degrees. This stress hazard point is a phenomenon not observed in thick-walled gears. Figure 9 shows the condition of the thin-walled gear after deformation.

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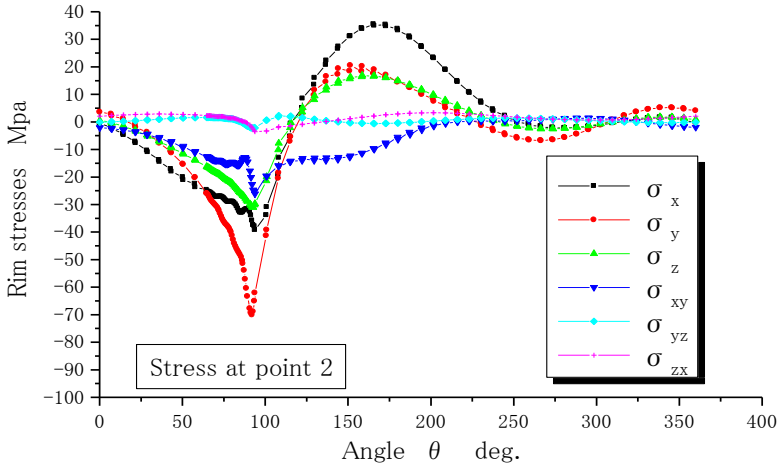
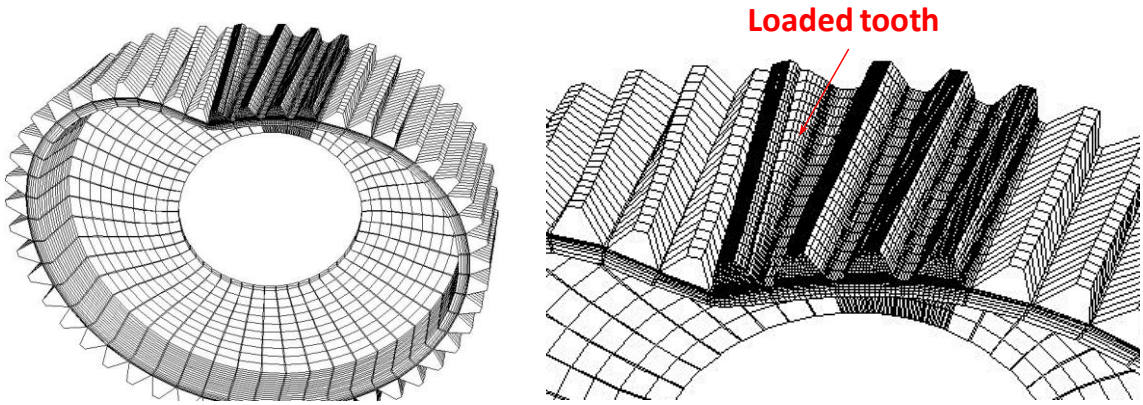


Figure 8: Stress distribution along the intersection circle of the rim and web



(a) Deformation of the entire gear　　(b)Close-up of dental deformity

Figure 9: Deformation of a gear and enlarged view of the deformation

References：

(1) Li Shuting, Ishida Takeshi, “A Method for Calculating Tooth Surface Loads on Thin-Walled Straight-Toothed Gears in Three-Dimensional Structures with Assembly Errors,” Transactions of the Japan Society of Mechanical Engineers (Series C), Vol. 63, No. 615, pp. 4017–4024, 1997

(2) Takeshi Ishida, Shu-Ting Li, “A Method for Analyzing Tooth Surface Loads and Contact Stresses in Three-Dimensional Thin-Walled Spur Gears Using the Finite Element Method and Linear Programming,” Transactions of the Japan Society of Mechanical Engineers (Series C), Vol. 63, No. 606, pp. 585–591, 1997

(3) Shuting Li, “Gear contact model and loaded tooth contact analysis of a three-dimensional, thin-rimmed gear”, Trans. ASME, Journal of Mechanical Design, Vol.124, Issue 3, pp.511-517, 2002

(4) Shuting Li, “Deformation and bending stress analysis of three-dimensional, thin-rimmed gears”, Trans. ASME, Journal of Mechanical Design, Vol.124, Issue 1, pp.129-135, 2002