文責：島根大学総合理工学部機械設計研究室教授　李　樹庭
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Design and Strength Analysis of Spur Gears with Errors and Tooth Modifications

By Prof. Shuting Li

1． Abstract

Tooth machining errors, assembly errors and modification have a significant impact on the tooth contact strength and bending strength of gears. Attempting to theoretically investigate how these factors specifically affect the contact strength of gear teeth requires sophisticated computer-based numerical analysis techniques; consequently, this issue has received very little research attention. To investigate this issue, around 1990, the author began research on the development of numerical analysis methods and software for analyzing the contact problem between a pair of gears using the three-dimensional (3D), finite element method (FEM). After many years of research, the author was finally able to resolve this problem. The results obtained are presented here; for further details, please refer to References[1–3].

2． Development of Design Calculation Software for a Pair of Spur Gears

To facilitate the design of gears, the author developed design calculation software for a pair of external spur gears, as shown in Figure 1. By entering the gear specifications into the “Input” section of this software and clicking the “Calculate” command, the results calculated by the software are displayed in the “Output” section. Furthermore, as shown in Figure 2, a drawing of the designed gear is output to an AutoCAD drafting template. Design software for internal gears has also been developed. Figure 3 shows an example of a designed internal gear.

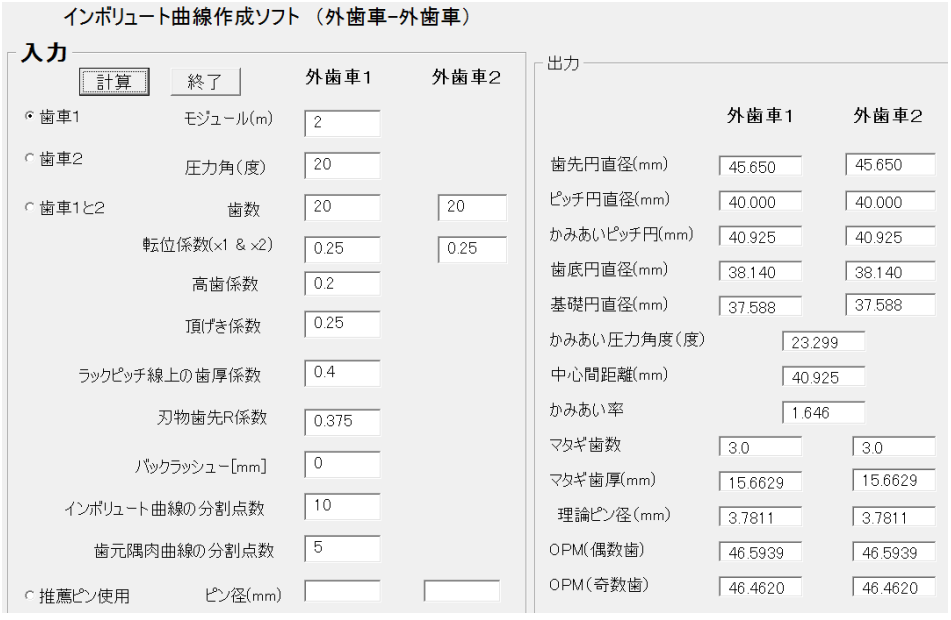

Figure 1: Design and calculation software for a pair of spur gears

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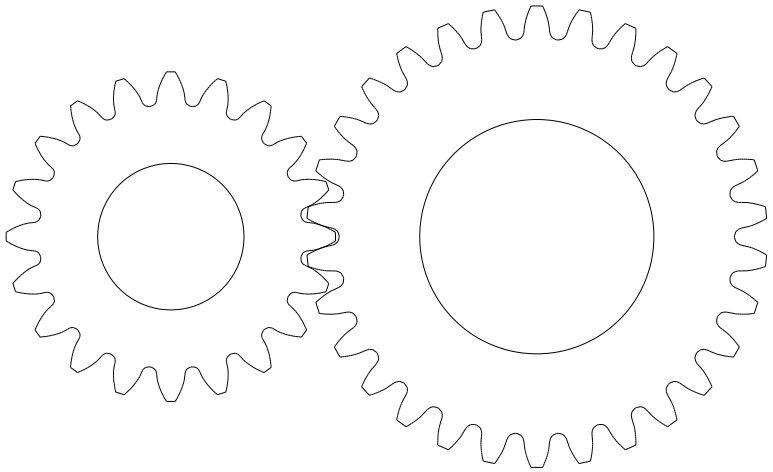


Figure 2: Meshing configuration of a pair of external spur gears (drawn using AutoCAD software)

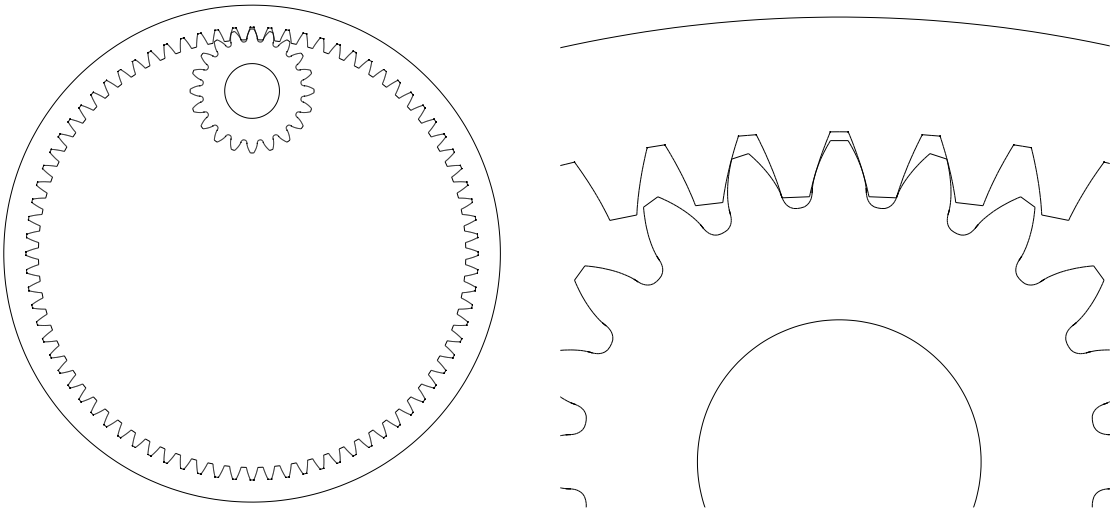


Figure 3: Meshing configuration of the internal spur gear (drawn using AutoCAD software)

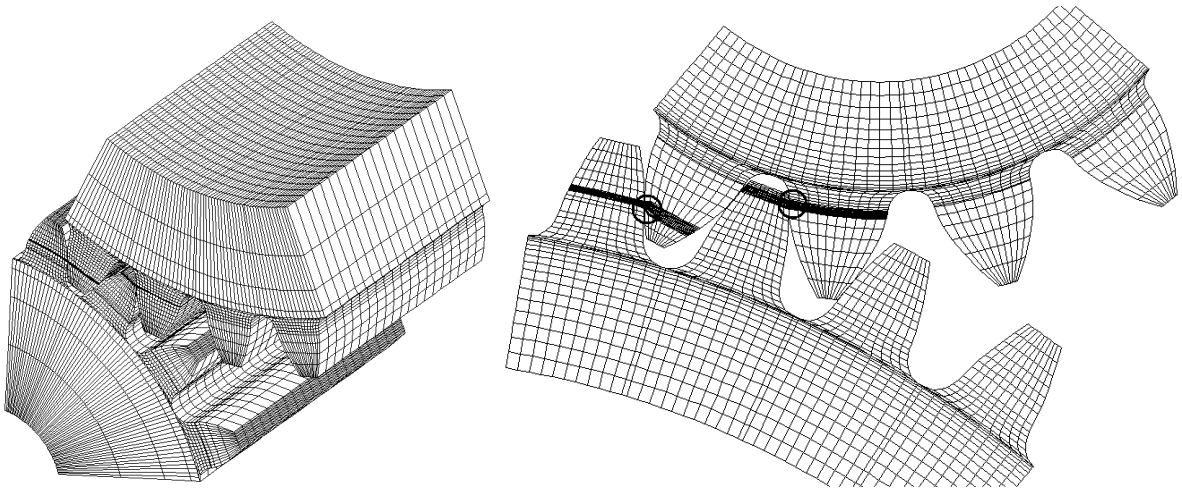
3． Numerical Analysis of the Contact Strength of a Pair of Cylindrical Spur Gears

Taking into account tooth surface machining errors, assembly errors and modification, the contact strength of a pair of spur gears was analyzed using specialized 3D, FEM software developed by the author. A brief overview of the tooth surface contact pressure calculated here is provided below; for details on root bending stress and the research, please refer to References[1–3].

Figure 4(a) shows the FEM model for the contact analysis of a pair of external spur gears, and Figure 4(b) illustrates the meshing configuration of the gears. Figure 5 shows the tooth surface contact region. As shown in Figure 5, the area hatched with red diagonal lines is the region used to analyze the tooth contact problem and is referred to as the contact region. The X-axis represents the tooth root direction, and the Y-axis represents the tooth profile direction.

Figure 6 is a contour plot of the tooth surface pressure distribution for an ideal gear (a gear without machining errors, assembly errors, or tooth profile modifications). Figure 7 shows the actual tooth surface

profile (tooth surface machining error) of a measured hobbed spur gear. When the gear's contact strength is analyzed using this tooth surface profile, the tooth surface pressure shown in Figure 6 yields the results shown in Figure 7. Therefore, it can be seen that tooth surface machining errors have a significant impact on the tooth surface pressure distribution.



(a) 3D-FEM model　　(b) Section view of the meshing teeth

Figure 4: 3D-FEM model for contact analysis of a pair of spur gears

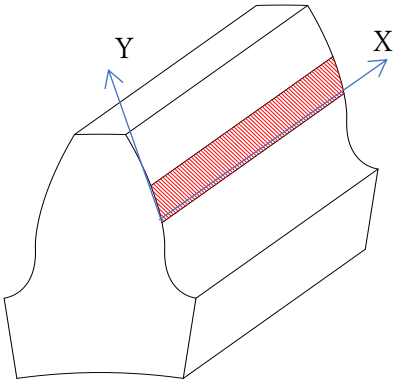


Figure 5: Area of contact analysis on the tooth surface (X-axis = tooth ridge; Y-axis = tooth profile)

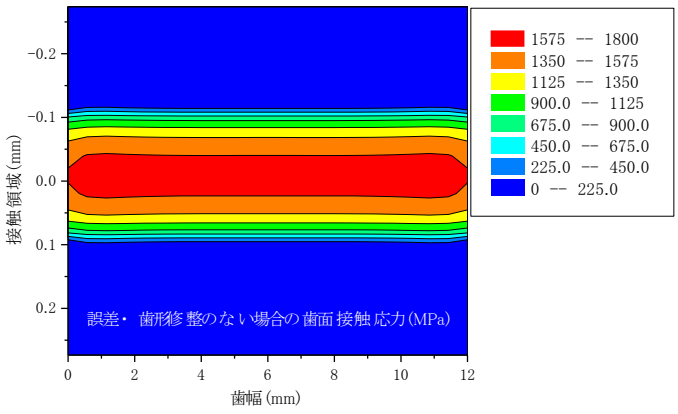


Figure 6: Contour plot of the tooth surface pressure distribution in the contact area of an ideal gear

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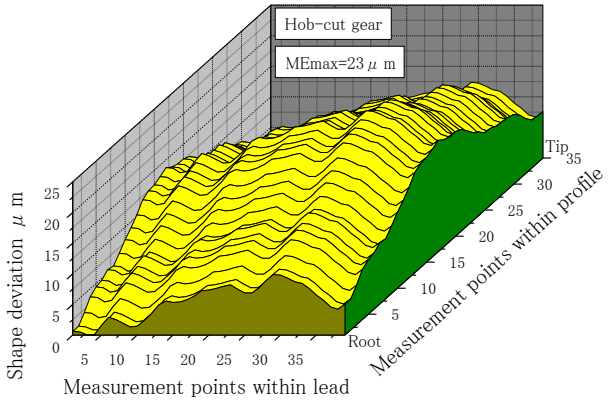

Figure 7: Tooth surface machining errors for hob-cut spur gears

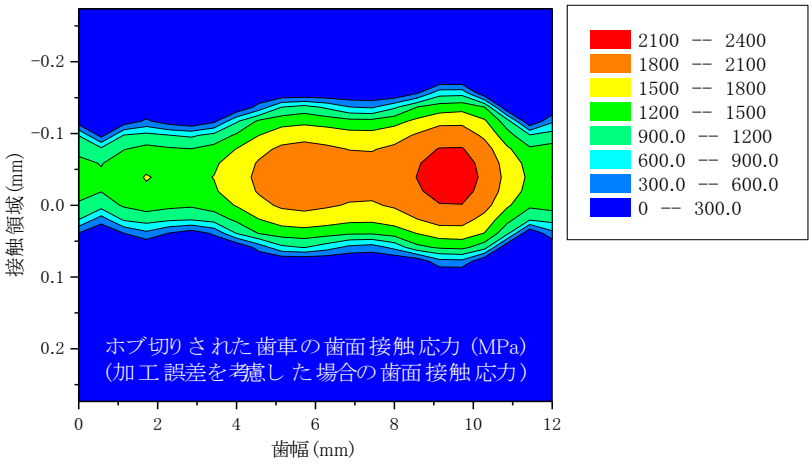

Figure 8: Contour map of tooth pressure distribution accounting for tooth machining errors

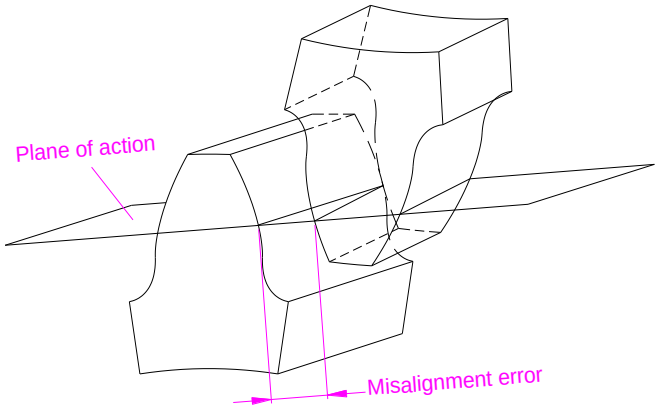

Figure 9: Tooth contact patterns when misalignment errors are present

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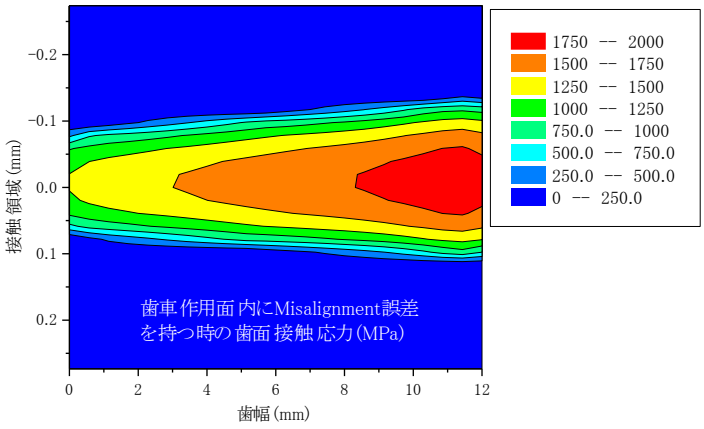

Figure 10: Contact pressure distribution on tooth surfaces when misalignment errors are present

Figure 9 shows the tooth contact pattern when there is a misalignment error in the gears. When the tooth contact strength was analyzed taking this misalignment error into account, the contact pressure on the tooth surfaces changed as shown in Figures 6 through 10.

When the tooth lead is crowned as shown in Figure 11, the contact pressure on the gear tooth surface changes as shown in Figures 6 through 12.

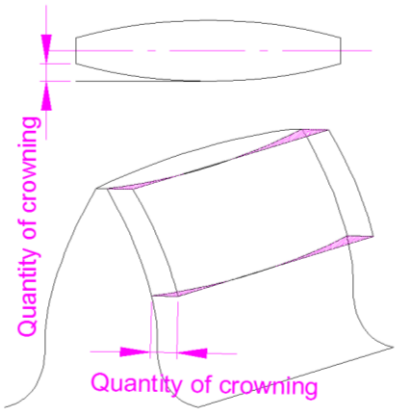

Figure 11　Tooth lead crowning

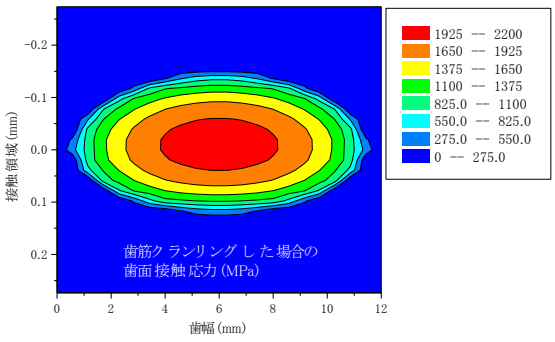

Figure 12: Contact pressure distribution on the tooth surface of a gear with crowning

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Figure 13 shows the modified tooth profile. As shown in Figure 13, modifying the tooth profile resulted in a change in the pressure distribution on the tooth surface, as illustrated in Figures 14 through 15. (The results from Figure 14 onward were analyzed using gears with different specifications.)

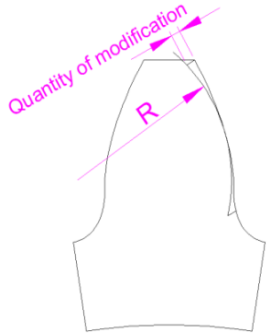

Figure 13: Tooth profile modification

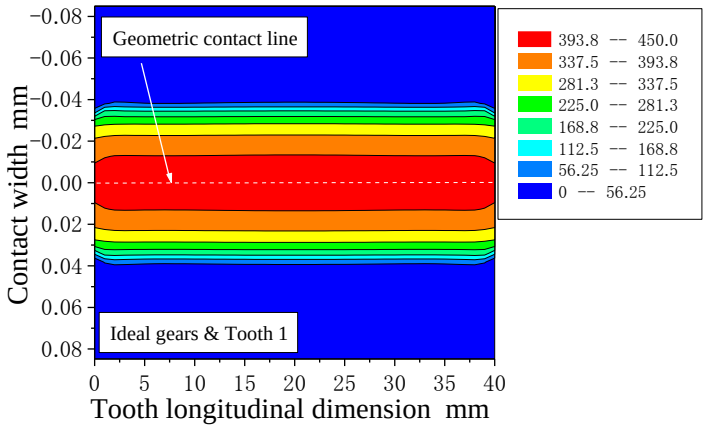

Figure 14: Contact pressure distribution on the tooth surface before tooth profile modification

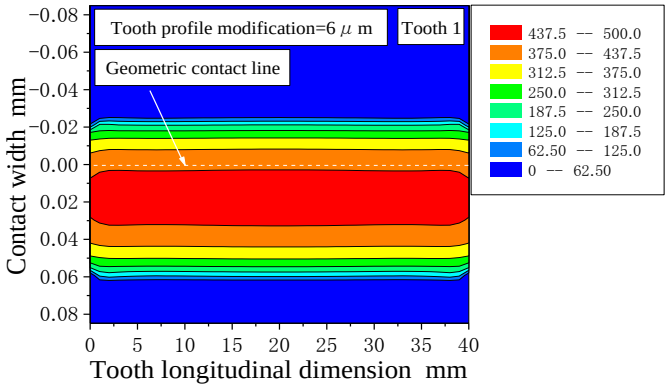

Figure 15: Contact pressure distribution on the tooth surface after tooth profile modification

Figure 16 shows the process of relieving the tooth flank. When relieving the tooth flank in a manner

similar to chamfering—that is, without rounding the intersection points between the relief line and the tooth flank line—the contact pressure on the tooth surface changes as shown in Figures 14 through 17. Figure 17 shows that edge loading occurs at the two points of intersection. To prevent the shape from becoming too sharp at these two points, the relieving line and the tooth flank line were smoothly connected using an arc; as a result, the tooth surface pressure changed as shown in Figures 17 through 18.

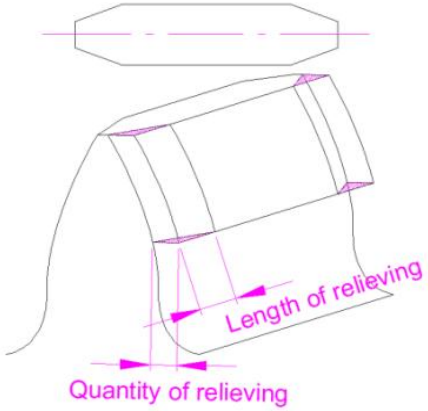

Figure 16: Tooth lead relief

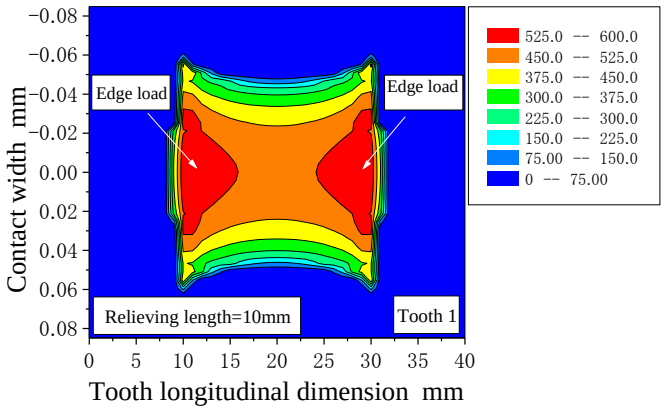

Figure 17: Contact pressure distribution on the tooth surface after tooth lead relief

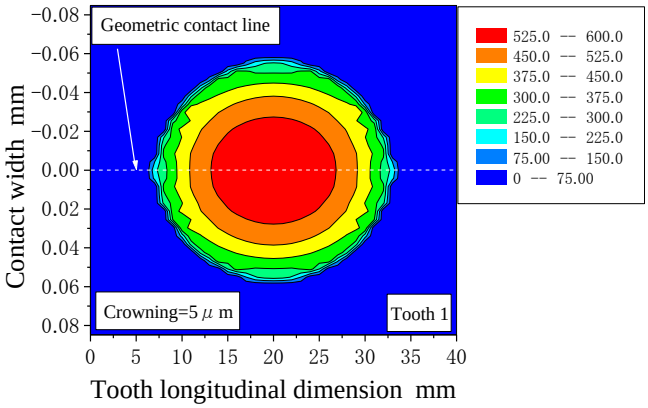

Figure 18: Contact pressure distribution on the tooth surface after tooth crowning

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References：

(1) Shuting Li, “Finite element analyses for contact strength and bending strength of a pair of spur gear with machining errors, assembly errors and tooth modifications”, Mechanism and Machine Theory, Elsevier Press, Vol.42, Issue 1, pp.88-114, 2007

(2) Shuting Li, “Effects of machining errors, assembly errors and tooth modifications on load-carrying capacity, load-sharing rate and transmission error of a pair of spur gear”, Mechanism and Machine Theory, Elsevier Press, Vol.42, Issue 6, pp.698-726, 2007

(3) Shuting Li, “Effects of misalignment error, tooth modifications and transmitted torque on tooth engagements of a pair of spur gears”, Mechanism and Machine Theory, Elsevier Press, Vol. 83, 2015, pp.125-136