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Design and Automatic Drafting Software Development of Planetary Gears

(In the case of using spur gears)

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1. Abstract

Planetary gear systems have a maximum gear ratio of 14 and are capable of transmitting high torque, making them widely used in machinery such as construction equipment and wind turbines. However, when designing a planetary gear system, complex calculations regarding tooth meshing are required to avoid tooth interference and to meet assembly constraints. To reduce design time and prevent calculation errors during the design process, these complex calculations must be performed using software. Therefore, the author developed design calculation software for planetary gear systems that can be used within AutoCAD. The following is a brief overview of the software's features.

2. Checking for Tooth Interference when Designing Planetary Gear Systems

Since internal gears are used in planetary gear sets, it is necessary to check for any meshing interference caused by the use of internal gears. There is a risk that the following three types of interference may occur, primarily in the meshing between internal and external gears, as well as in the meshing between internal gears and the pinion cutter used for cutting them [1–3].

（1）Involute interference：

Involute interference refers to the interference phenomenon that occurs when the root of the external gear meshes with the tip of the internal gear. Generally, this interference is more likely to occur when the external gear has fewer teeth. The conditions under which this interference does not occur are given by Equation (1.1).

$$\frac{z_1}{z_2} \geq 1 - \frac{\tan \alpha_{a2}}{\tan \alpha_w} \tag{1.1}$$

Where,Z_1=the number of teeth on the external gear；　Z_2= Number of teeth on an internal gear；
α_{a2}＝Tooth tip pressure angle of an internal gear；　α_w＝Contact Pressure Angle
These contact angles are calculated using Equations (1.2) and (1.3), respectively.

$$\alpha_{a2} = \cos^{-1}\left(\frac{d_{b2}}{d_{a2}}\right) \tag{1.2}$$

$$\alpha_w = \cos^{-1}\left(\frac{(z_2 - z_1)m\cos\alpha}{2\boldsymbol{a}}\right) \tag{1.3}$$

Where, m= Gear Module;　d_{b2}= Base Circle Diameter of an Internal Gear；　d_{a2}= Pitch circle diameter of a internal gear;　$\boldsymbol{a}$= Center-to-center distance of gears；　α= Angle of Pressure on the Reference Pitch

Circle（α=20°）.

For Equation (1.1) to hold, the tip circle of the internal gear must be larger than the base circle. Thus, Equation (1.4) is obtained.

$$d_{a2} \geq d_{b2} \tag{1.4}$$

For a standard internal gear with a reference pressure angle $\alpha = 20°$, the tip circle of the internal gear will not be larger than the reference circle unless $z_2 > 34$; therefore, when designing a standard internal gear, the number of teeth must be 34 or more.

（2）Trochoidal Interference

Trochoidal interference refers to the interference that occurs between the tooth tips of the external gear and the internal gear when the tooth tip of the external gear exits the tooth slot of the internal gear. This phenomenon occurs when the difference in the number of teeth between the internal and external gears is small. The condition for preventing this interference is given by Equation (2.1).

$$\theta_1 \frac{z_1}{z_2} + \text{inv}\ \alpha_w - \text{inv}\ \alpha_{a2} \geq \theta_2 \tag{2.1}$$

Where, θ_1 and θ_2 are calculated using Equations (2.2) and (2.3), respectively.

$$\theta_1 = \cos^{-1}\left(\frac{{r_{a2}}^2 - {r_{a1}}^2 - a^2}{2ar_{a1}}\right) + \text{inv}\ \alpha_{a1} - \text{inv}\ \alpha_w \tag{2.2}$$

$$\theta_2 = \cos^{-1}\left(\frac{a^2 + {r_{a2}}^2 - {r_{a1}}^2}{2ar_{a2}}\right) \tag{2.3}$$

Where, r_b and r_c are the radii of the tip circles of the spur gear and the internal gear, respectively. Furthermore, α_{a1} is the tip pressure angle of the spur gear, which is given by the following equation.

$$\alpha_{a1} = \cos^{-1}\left(\frac{d_{b1}}{d_{a1}}\right) \tag{2.4}$$

Where, α_{a1}= Tooth Tip Pressure Angle in External Gears；　r_{a2}= Tip radius of an internal gear；
r_{a1}= Tip radius of an external gear；　d_{b1}= Base circle diameter of an external Gear；
d_{a1}= Pitch circle diameter of an external gear；　a= Center-to-center distance of gears.

For a standard gear with a standard pressure angle $\alpha = 20°$, trochoidal interference does not occur if the difference in the number of teeth $(Z_2 - Z_1)$ is 9 or greater.

（3）Cropping Interference

This is also known as cutter clearance interference. When cutting the teeth of an internal gear with a pinion cutter, if the pinion cutter cuts into the internal gear in the radial direction, the cutting edge of the pinion cutter may cut off the tip of the internal gear's tooth. In a gear pair, this phenomenon occurs when, even if the smaller gear is inserted axially into the internal gear, it becomes impossible to move the smaller gear radially away from the center of the internal gear to engage the teeth. This phenomenon is observed

when the difference in the number of teeth between the internal gear and the pinion cutter is small. To prevent this phenomenon from occurring, the following equation must be satisfied.

$$\theta_1 + \text{inv}\ \alpha_{a1} - \text{inv}\ \alpha_w \geq \frac{Z_c}{Z_b}(\theta_2 + \text{inv}\ \alpha_{a2} - \text{inv}\ \alpha_w) \tag{3.1}$$

Where, θ_1 and θ_2 are calculated using Equations (3.2) and (3.3), respectively.

$$\theta_1 = \sin^{-1}\sqrt{\frac{1-\left(\cos\alpha_{a1}/\cos\alpha_{a2}\right)^2}{1-\left(Z_1/Z_2\right)^2}} \tag{3.2}$$

$$\theta_2 = \sin^{-1}\sqrt{\frac{1-\left(\cos\alpha_{a2}/\cos\alpha_{a1}\right)^2}{\left(Z_2/Z_1\right)^2-1}} \tag{3.3}$$

3. Assembly Constraints for Planetary Gear Sets

To ensure the proper functioning of the planetary gear mechanism, the following three constraints are imposed [1–3].

（1）Coaxial Constraints

For a planetary gear train to function properly, the center-to-center distance between the planetary gears and the internal gear when they mesh must be equal to the center-to-center distance between the sun gear and the planetary gears when they mesh. This condition is known as the coaxiality constraint for planetary gear trains. When standard internal and external spur gears without displacement are used in a planetary gear train, this coaxiality constraint can be simply expressed by Equation (4.1) as shown in Figure 1.

$$\begin{aligned}&\text{Reference pitch circle radius of a sun gear} + 2 \times \text{Pitch circle radius of a planetary gear}\\ &= \text{Reference pitch circle radius of an internal gear}\end{aligned} \tag{4.1}$$

Let the pitch circle radii of the sun gear, planet gear, and internal gear be r_S, r_P, and r_I, respectively; then, Equation (4.1) can be rewritten as Equation (4.2). Furthermore, using the relationship that the pitch circle diameter of a gear equals the number of teeth multiplied by the module, Equation (4.3) is obtained. Rearranging Equation (4.3) yields Equation (4.4). That is, when determining the number of teeth for the sun gear, planet gears, and internal gear, Equation (4.4) must be satisfied. Here, m is the module of the gear.

$$r_s + 2r_P = r_I \tag{4.2}$$

$$\frac{1}{2}mZ_s + 2 \times \left(\frac{1}{2}mZ_P\right) = \frac{1}{2}\text{m}Z_I \tag{4.3}$$

$$Z_s + 2 \times Z_P = Z_I \tag{4.4}$$

In the case of a planetary gear mechanism using offset gears, the reference pitch circle shown in Equation

(4.1) becomes the meshing pitch circle; however, the calculation is simplified if you verify that “the distance between the axes of the planetary gear and the internal gear equals the distance between the axes of the planetary gear and the sun gear.”

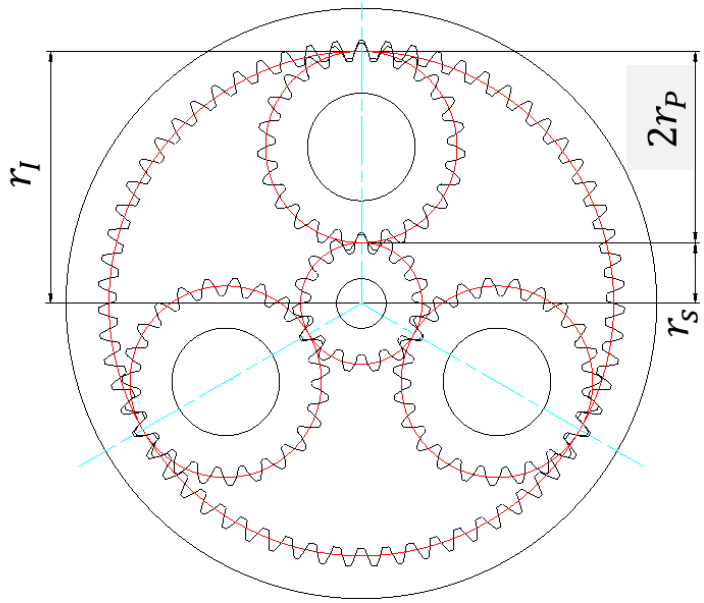

Figure 1: Diagram Illustrating Coaxial Constraints

（２）Constraints on the Arrangement of Planetary Gears

As shown in Figure 2, in order to arrange the three planetary gears at equal circumferential intervals, the closed thick line—which consists of arc segments on the pitch circles of the internal gear, the planetary gears, and the sun gear—must be an integer multiple of the tooth’s circumferential pitch. That is, the value obtained by dividing (the length of the partial arc L_1 on the pitch circle of the inner gear + 2 times the length of the partial arc − on the pitch circle of the planetary gear + the length of the partial arcL_3 on the pitch circle of the sun gear) by the circumferential pitch of the teeth must be an integer. The partial arc length on the pitch circle of the internal gear is$L_1 = mZ_I\pi/n$, the partial arc lengthL_2 on the pitch circle of the planetary gear is $L_2 = mZ_P\pi/2$, the partial arc length L_3 on the pitch circle of the sun gear is$L_3 = mZ_S\pi/n$, and the tooth pitch is $t = m\pi$. Therefore, the configuration constraints for the planetary gears can be expressed by Equation (5.1).

$$\frac{mZ_I\pi/n + 2 \times mZ_P\pi/2 + mZ_S\pi/n}{m\pi} = \frac{1}{n}(Z_I + Z_S) + Z_P = \text{Integer} \qquad (5.1)$$

Furthermore, since the number of teeth Z_P on a planetary gear is always an integer, Equation (5.1) can be simplified to Equation (5.2). Equation (5.2) represents the configuration constraints for planetary gears.

$$\frac{1}{n}(Z_I + Z_S) = \text{Integer} \qquad (5.2)$$

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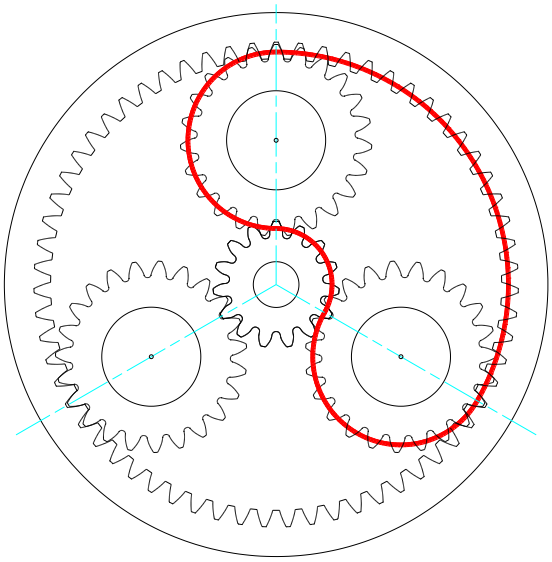

Figure 2: Layout Constraints for Planetary Gears

（3）Clearance Requirements for Planetary Gears

As shown in Figure 3, when using a large number of planetary gears, it is necessary to impose the constraint in Equation (6.1) to prevent adjacent planetary gears from colliding with one another.

(Tip Circle Radius of Planetary Gear 1 + Tip Circle Radius of Planetary Gear 2) < Center-to-center distance L between adjacent planetary gears	(6.1)

Since the N planetary gears are designed with exactly the same specifications, the left-hand side of the above equation represents the diameter of the tip circle of the planetary gears. That is, Equation (6.1) can be rewritten as Equation (6.2).

The tip circle diameter of a planetary gear < the center-to-center distance L between adjacent planetary gears (6.2)

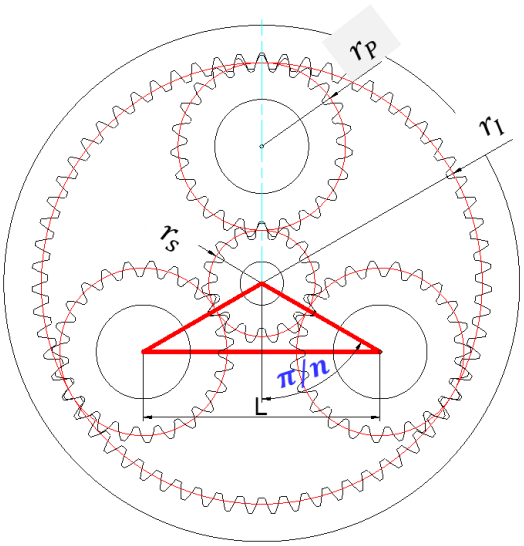

Figure 3: Diagram illustrating the clearance requirements between planetary gears

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When using a standard gear without displacement, the tip circle diameter of a planetary gear can be calculated using Equation (6.3). Furthermore, since the center-to-center distance L between adjacent planetary gears is calculated using Equation (6.4) for the isosceles triangle shown in Figure 3, substituting Equations (6.3) and (6.4) into Equation (6.1) yields Equation (6.5). Equation (6.5) is also known as the external interference condition for planetary gears.

$$2 \times (0.5mZ_P + h_k) = 2 \times (0.5mZ_P + m) \tag{6.3}$$

$$L = 2\frac{1}{2}m(Z_s + Z_P)\sin\frac{\pi}{n} \tag{6.4}$$

$$Z_P + 2 < (Z_S + Z_P)\sin\frac{\pi}{n} \tag{6.5}$$

4. Development of Design Calculation Software for Planetary Gear Systems

Figure 4 shows the main screen of the design calculation software developed for planetary gear systems, and Figure 5 shows the planetary gear system output to an AutoCAD drafting template following the design calculation. With this software, designing planetary gear systems becomes much easier.

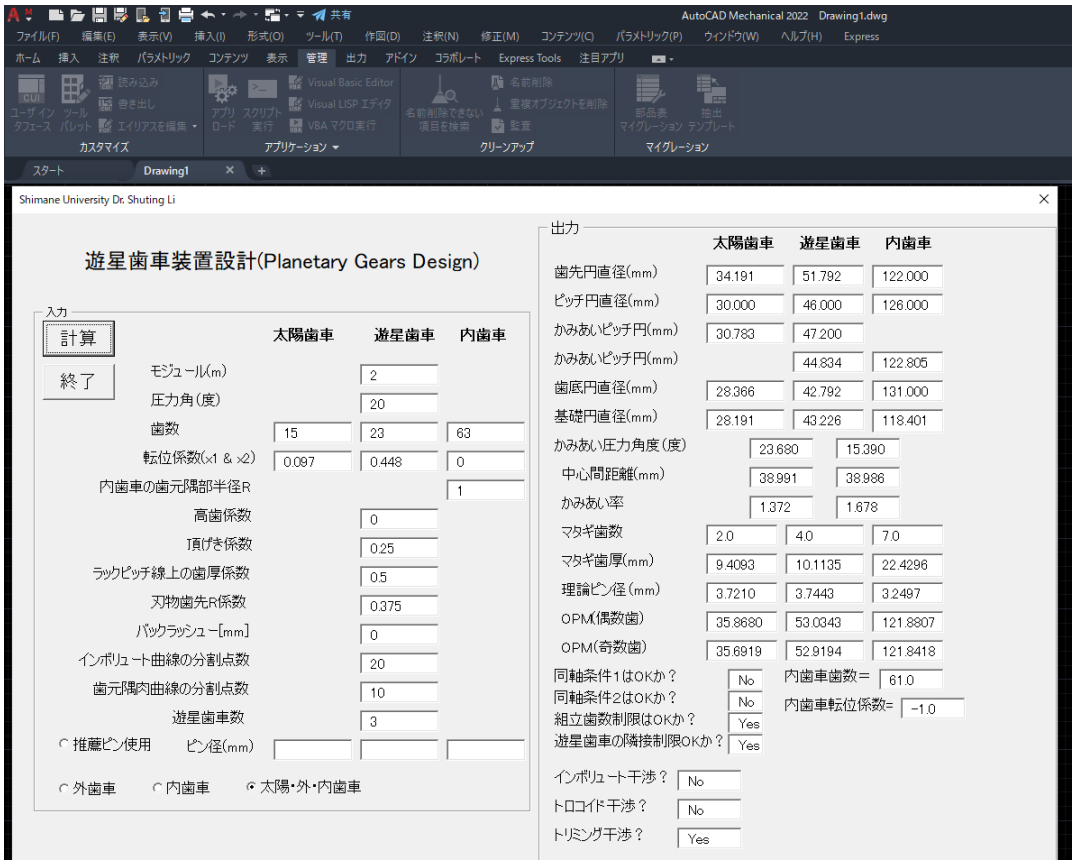

Figure 4: Main screen of the design calculation software developed for the planetary gear system

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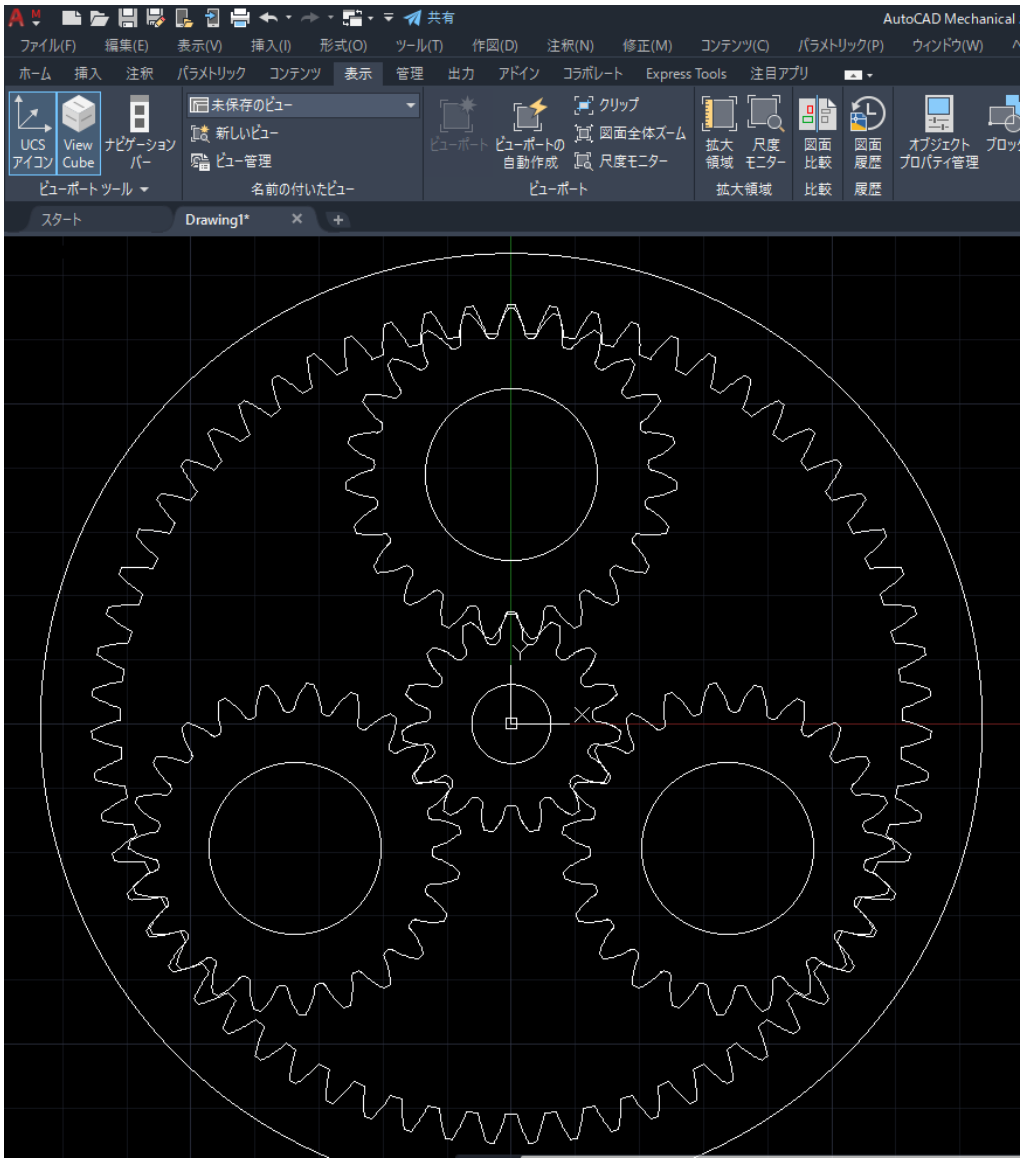

Figure 5: View of the planetary gear set output to an AutoCAD drafting template following design calculations

References：

(1) Ohara Gear Industry Co., Ltd., Gear Technical Data
(2) Ohara Gear Industry Co., Ltd., Product Catalog
(3) Kyouiku gear industry Co., Ltd., Product Catalog