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A Strength Calculation Method of Involute Spline Couplings

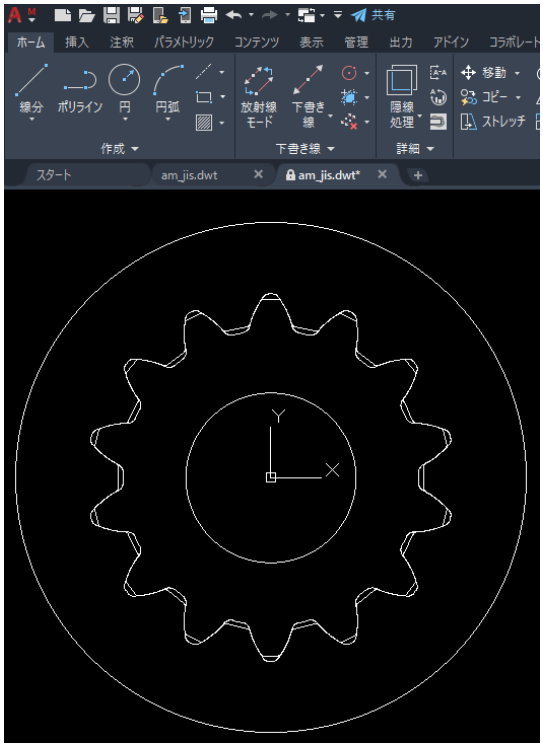
By Prof. Shuting Li

1． Abstract

Since involute splines can transmit high torque, they are commonly used to connect shafts to shafts or shafts to gears instead of keys. However, because it is difficult to calculate the strength of an involute spline after it has been designed, the author, through many years of research, proposed a three-dimensional finite element method for numerically calculating the strength of involute splines and developed dedicated software. Here, we briefly introduce the content and results of this research. For further details on this research, please refer to Reference [1].

2． Development of Design Software for Involute Splines

We developed design software for involute splines to reduce the time required for their design. This software was developed to be integrated into AutoCAD and can be used within AutoCAD. Figure 1(a) shows the input screen of the software, and Figure 1(b) shows the calculation results for an involute spline designed using the software. After entering the gear parameters required to design the involute spline in Figure 1(a) and clicking the “Calculate” button, the calculation results shown in Figure 1(b) are displayed immediately.



(a)　Software Input Screen　　(b) Software Calculation Results

Figure 1: Development of design software for involute splines

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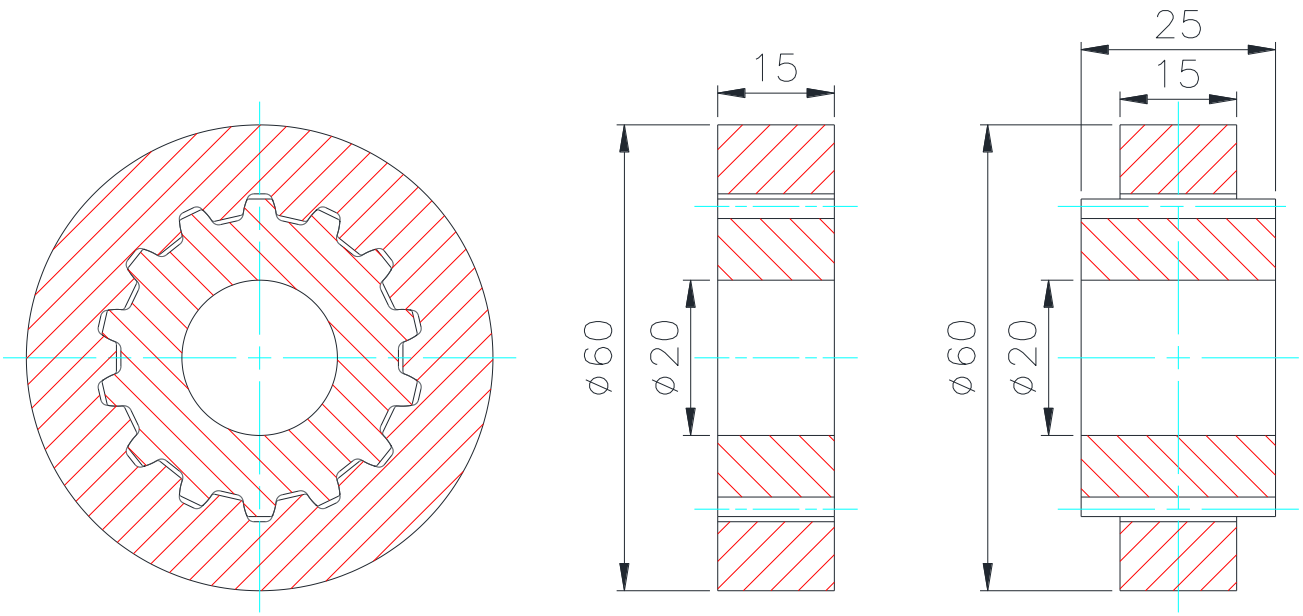
3． Development of Software for Strength Analysis of Involute Splines

Table 1 lists the specifications of the involute spline under study. Figure 2 shows its cross-sectional shape. Figure 2(b) shows the case where the tooth widths are the same, while Figure 2(c) shows the case where the tooth widths differ. The strength analysis of the involute spline was performed using the dedicated three-dimensional finite element method described in Reference [1] and the software developed for this purpose. A detailed explanation of this method and the software is omitted here; only the analysis results are presented below.

Table 1 Gearing parameters of the involute spline couplings used in this paper.

Items	Symbol	Results
Tooth Profile		Involute
Number of teeth	Z	14
Module	m	2.5 (mm)
Pressure angle	α_c	20 (degrees)
Shifting coefficient	x	+0.8
Edge radius of cutter	r_c	0.375m
Addendum		$0.4m$ (mm)
Clearance		$0.25m$ (mm)
Face width of the hub	b	20 (mm)
Diameter of the meshing pitch circle	D_m	$D_m = (Z + 2x)m$
Loaded torque	T	411.6 (Nm)



(a) Cross-sectional Shape of a Tooth (b) When the tooth widths are the same (c) When tooth widths differ

Figure 2: Involute spline couplings under study

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4． Strength Analysis Results for Involute Splines with the Same Tooth Width

To analyze the strength of involute splines with the same tooth width shown in Figure 2(b), we used the 3D-FEM model of the involute spline shown in Figure 3. Figure 4 shows the contour plot of the tooth surface pressure distribution analyzed using this model and the dedicated 3D finite element method software we developed; Figure 5 shows the tooth surface pressure distribution along the tooth root direction; Figure 6 shows the tooth surface pressure distribution along the tooth profile direction; Figure 7 shows the distribution of bending stress along the tooth root at the point of maximum bending stress, and Figure 9 shows the contour plot of the shear stress distribution on the cylindrical surface at the point of maximum shear stress at the tooth root. The cylindrical surface at the point of maximum shear stress at the tooth root is defined as shown in Figure 8.

Table 2 compares the results obtained using the dedicated 3D finite element method and Approximation Method (1).

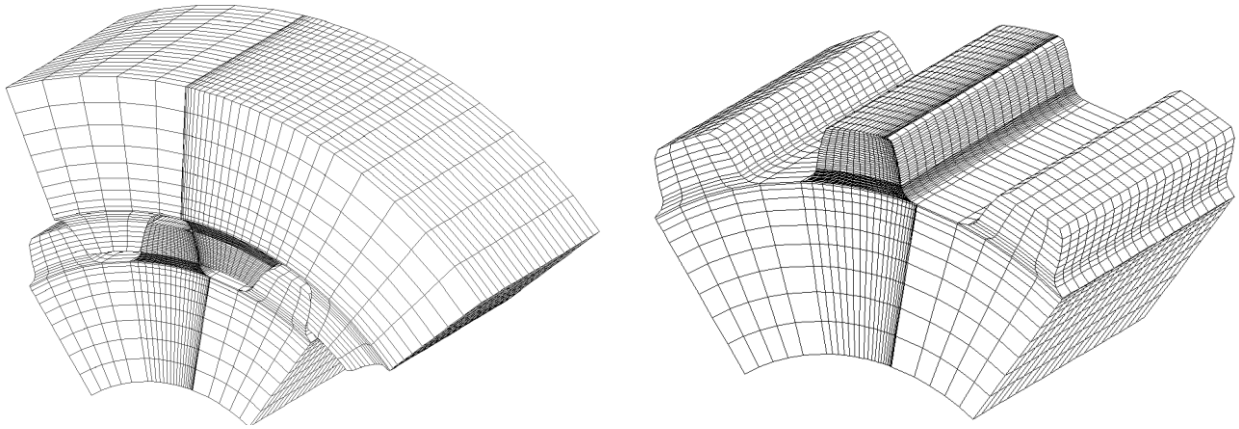

Figure 3: FEM model for contact analysis

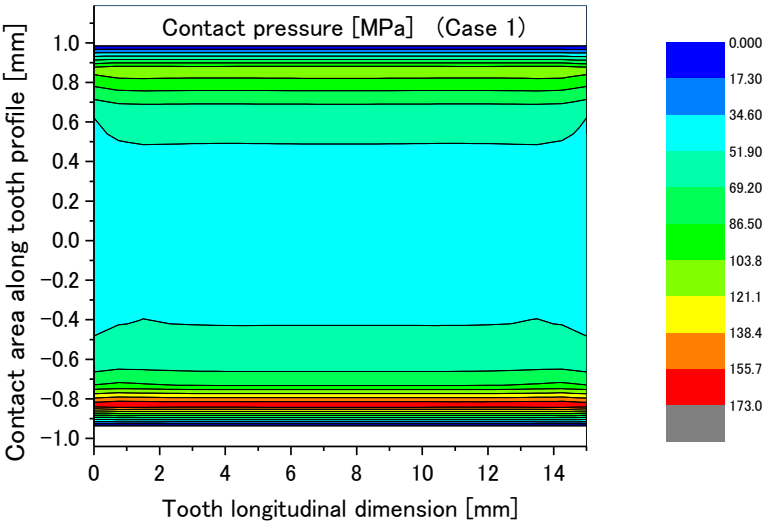

Figure 4: Contour map of surface pressure distribution on a tooth surface

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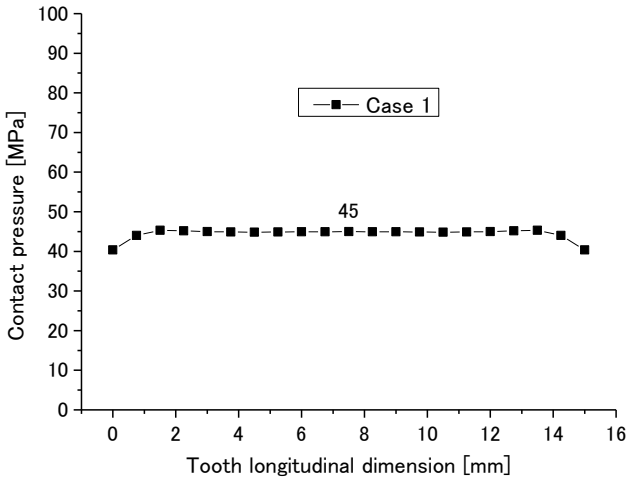

Figure 5: Surface pressure distribution along the tooth groove direction

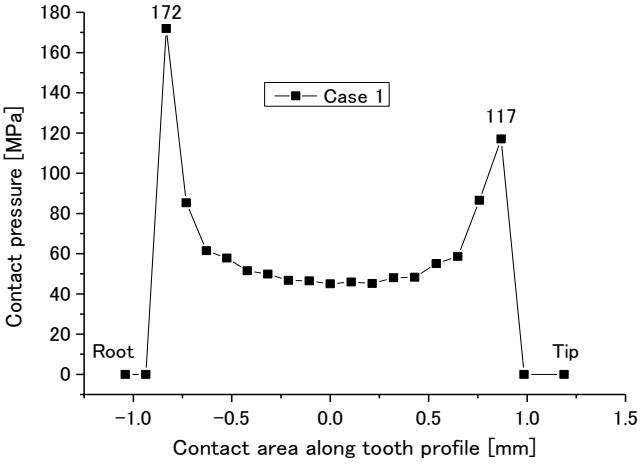

Figure 6: Surface pressure distribution along the tooth profile

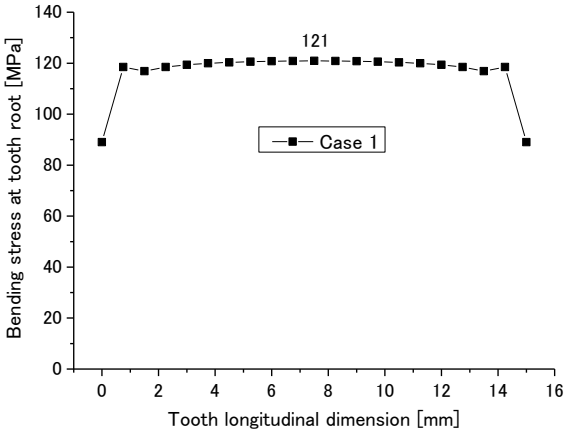

Figure 7: Distribution of bending stress at the tooth root along the tooth ridge direction

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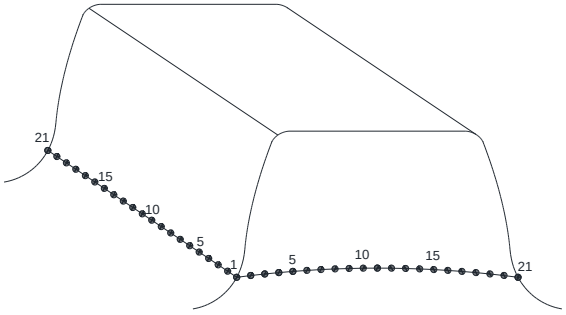

Figure 8: Definition of the cylindrical surface of maximum shear stress at the tooth root

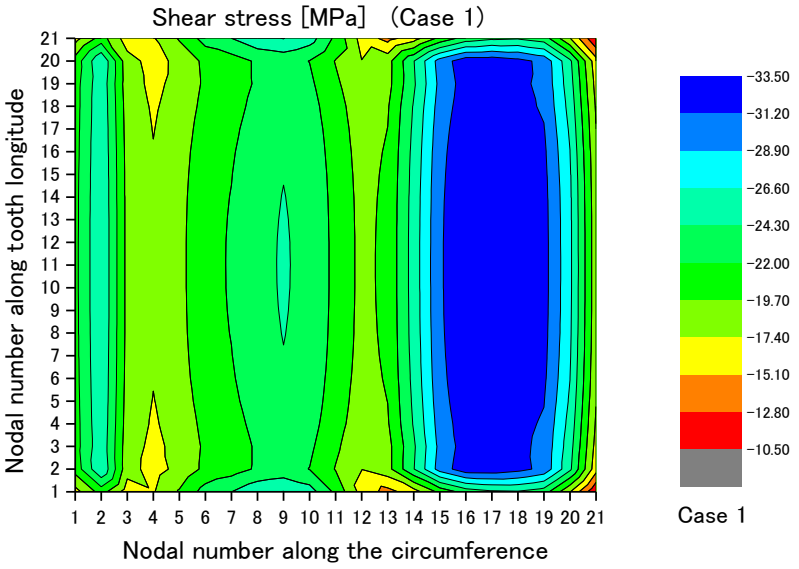

Figure 9: Shear stress distribution on a cylindrical surface at the point of maximum shear stress at the tooth root

Table 2 Comparison between the FEM software and the approximation method

The maximum stresses	Developed FEM software	Approximation method
Tooth contact stress (MPa)	172	50.3
Shear stress at tooth root (MPa)	33.5	25.6
Bending stress at tooth root (MPa)	121	63.5

5． Strength Analysis Results for Involute Splines with Different Tooth Widths

To analyze the strength of involute splines with different tooth widths, as shown in Figure 2(c), we used the 3D FEM model of the involute spline shown in Figure 10. Figure 11 shows the contour plot of the tooth surface pressure distribution analyzed using this model and the dedicated 3D finite element method software we developed; Figure 12 shows the tooth surface pressure distribution along the tooth root direction; Figure 13 shows the tooth surface pressure distribution along the tooth profile direction; Figure 14 shows the distribution of bending stress along the tooth root at the point of maximum bending stress,

and Figure 15 shows the contour plot of the shear stress distribution on the cylindrical surface at the point of maximum shear stress at the tooth root. The cylindrical surface at the point of maximum shear stress at the tooth root is defined as shown in Figure 8.

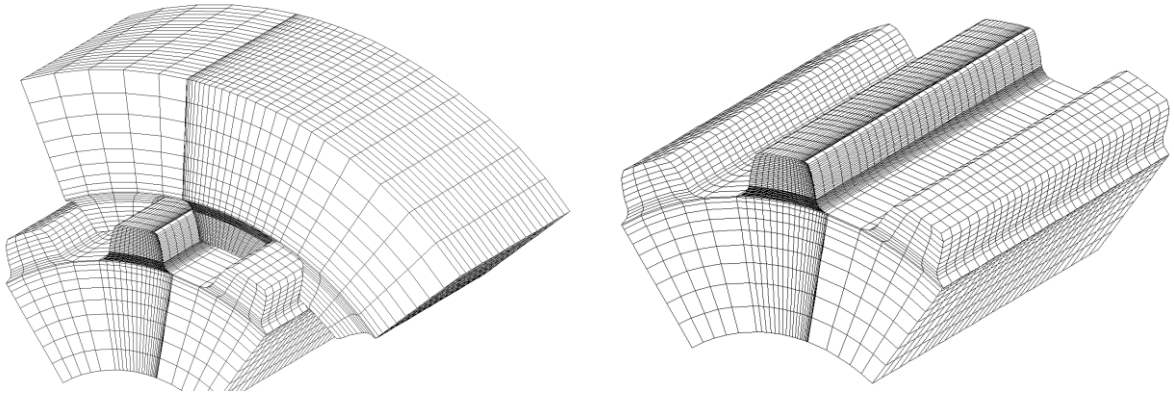

Figure 10: FEM model for contact analysis

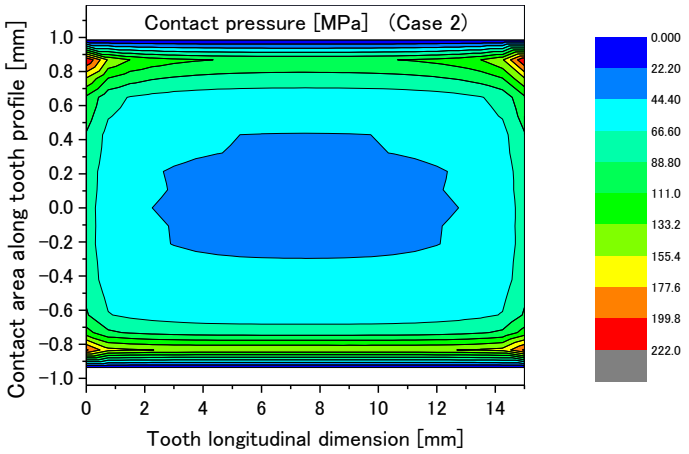

Figure 11: Contour map of surface pressure distribution on a tooth surface

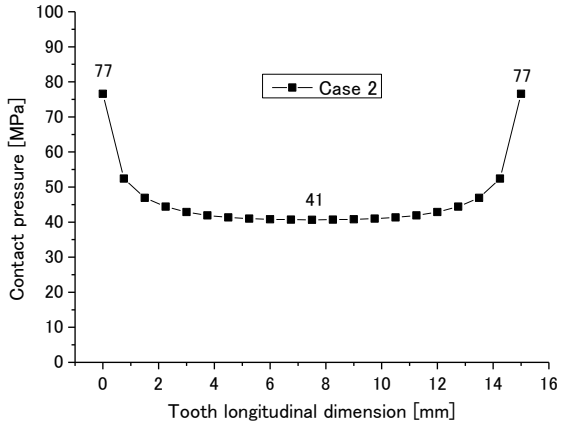

Figure 12: Surface pressure distribution along the direction of the tooth groove

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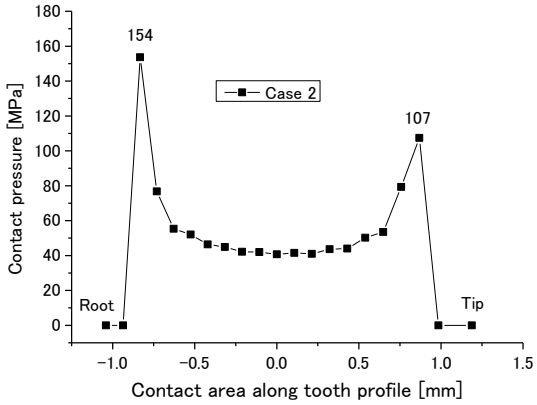

Figure 13: Surface pressure distribution along the tooth profile direction

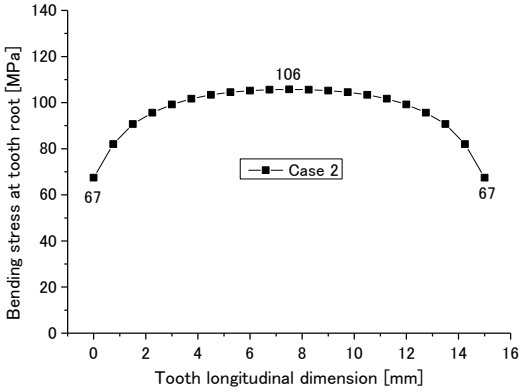

Figure 14: Distribution of bending stress at the tooth root along the tooth flank direction

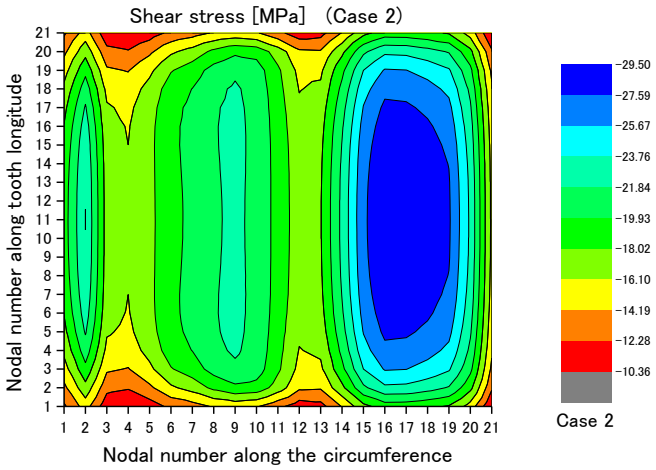

Figure 15: Shear stress distribution on a cylindrical surface at the point of maximum shear stress at the tooth root

References：

(1) Shuting Li, “Contact analysis and strength calculations of involute spline couplings”, Scientific Reports 13, 07 January 2023, Springer Nature, pp.1-22, https://10.1038/s41598-023-27615-2